University of Glasgow

ASTRONOMY & PHYSICS EDUCATION

Advanced Higher Physics: Assignment Support

Astronomy & Physics Education Group

School of Physics & Astronomy

University of Glasgow

Measurement of wavelength by Michelson Interferometer

Introduction

A Michelson Interferometer creates interference patterns by taking a single source of light, splitting that light into two different paths and then recombining them. A schematic diagram showing the standard form of a Michelson Interferometer is shown in Figure 1. S is the source of the light, which is diffused by screen D and then split into two parts by partial reflection at the beam splitter, BS. The resulting two light beams then follow different paths (vertically and horizontally in the Figure) and, after reflection at M_1 and M_2, are brought together again to produce interference fringes. The compensator plate, C, is used to equalise the path lengths in glass of the two light beams.

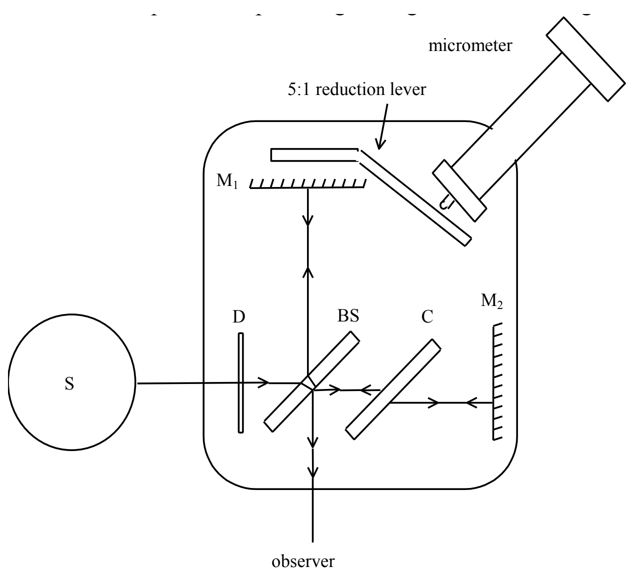

Figure 1: Schematic diagram of a Michelson interferometer.

In this particular make of interferometer, movement of mirror M_1 is controlled via a level arm, which has a reduction ratio of 5:1 – i.e. the lever makes the mirror M_1 move only a fifth of the distance moved by the micrometer tip. The value of the reduction ratio is approximate – determination of the exact ratio is need before detailed experiments can be carried out.

Circular fringes are produced using monochromatic light when the mirrors M_1 and M_2 are exactly perpendicular. The formation of these fringes may be more readily understood by considering Figure 2 which illustrates the essential features of the interferometer. Here the interferometer has been "unfolded". In this figure, M_2 has been replaced by M'_2, the reflection of M_2 in the beam splitter, as seen by the observer.

Due to reflection in the real interferometer the source S appears to be behind the observer who sees two virtual images of S, S_1 and S_2 reflected by M_1 and M'_2. If M_1 is parallel to M'_2 and if the separation of M_1 and M'_2 is d, then the distance S_1 to S_2 will be $2d$.

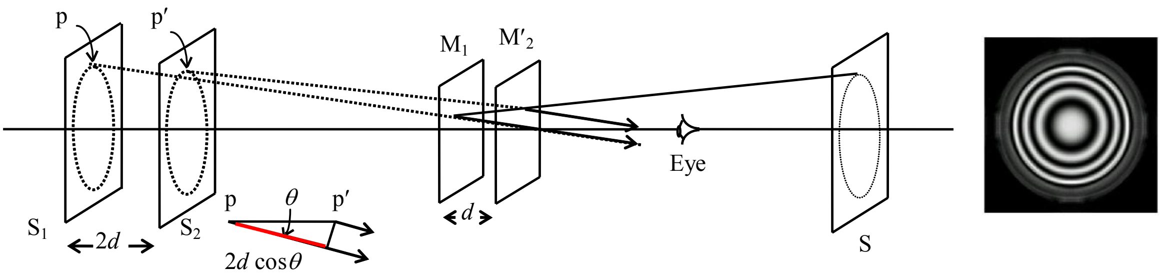

Figure 2:

If the incident light has wavelength λ, then for $2d = m\lambda$ (where m is an integer) all light reflected normal to the mirrors will be in phase. However, rays reflected at an angle θ will not, in general, be in phase.

The path differences from rays coming to the eye (focused to receive parallel light) from corresponding points p and p' (as shown in red in Figure 2) is $2d\cos(\theta)$. Thus the rays will reinforce and produce a maximum (assuming there are no phase changes introduced by the reflections of the light beams) when

$$2d\cos(\theta) = m\lambda \quad [1]$$

For parallel mirrors, the set up is cylindrically symmetric. When you look at the interferometer, the rays will be at a slight angle to your eye, therefore you will see circles. If M_1 is slowly moved inwards, those circular fringes collapse inwards and vanish at the centre axis, where $\cos(\theta) = 1$ and hence [1] simplifies to

$$2d = m\lambda$$

[2]

A circular fringe will disappear each time $2d$ decreases by λ, or equivalently when d decreases by $\frac{\lambda}{2}$. Conversely, if M_1 is moved outwards, therefore increasing the distance d, the circular fringes will emerge from the centre and expand, a circular fringe emerging each time $2d$ increases by λ, or equivalently when d increases by $\frac{\lambda}{2}$.

Using monochromatic light, the circular fringes are visible for very long path differences in the interferometer. If the source consists of two closely spaced lines of wavelengths λ_1 and λ_2, where $\lambda_1 > \lambda_2$ (such as when using the Sodium Doublet) the "visibility" of the fringe pattern fluctuates regularly through highly contrasted maxima and vanishings of the fringe pattern as the path difference is changed. This fluctuation is caused by the two fringe patterns corresponding to the two wavelengths getting in and out of step as d varies. With circular fringes the path difference can be adjusted until the pattern appears to vanish, at least near the centre of the pattern. This first occurs when the fringe numbers for the two wavelengths differ by $\frac{1}{2}$ because then the centre of the fringe pattern for one wavelength is light when that for the other is dark, leading to minimum visibility. If this first vanishing of the fringe pattern occurs when the mirror separation is d then the fringe numbers are $\frac{2d}{\lambda_1}$ and $\frac{2d}{\lambda_2}$ respectively and so

$$\Delta m = \frac{2d}{\lambda_2} - \frac{2d}{\lambda_1} = \frac{1}{2}$$

[3]

If the separation of the mirrors is now increased by an amount δd until the next vanishing of the fringe pattern is obtained then this will occur when the difference in fringe number becomes 3/2 and so ...

$$\Delta m = \frac{2(d + \delta d)}{\lambda_2} - \frac{2d(d + \delta d)}{\lambda_1} = \frac{3}{2}$$

[4]

[4]-[3] then gives us ...

$$\frac{2\delta d}{\lambda_2} - \frac{2\delta d}{\lambda_1} = 1$$

$$\Rightarrow \ \delta d = \frac{\lambda_1 \lambda_2}{2(\lambda_1 - \lambda_2)}$$

[5]

If a doublet of known wavelengths, e.g. the mercury orange doublet, is used then δd can be determined using [5].

Conversely, if the wavelengths are relatively close in value (i.e. $\lambda_1 \approx \lambda_2$, we can calculate an average wavelength,

$$\lambda_{ave} = \frac{\lambda_1 + \lambda_2}{2}$$

and then re-write [5]:

$$\delta d = \frac{\lambda_1 \lambda_2}{2(\lambda_1 - \lambda_2)} = \frac{\lambda_{ave}^2}{2(\lambda_1 - \lambda_2)}$$

$$\boxed{\Rightarrow (\lambda_1 - \lambda_2) = \frac{\lambda_{ave}^2}{2\delta d}}$$

[6]

This allows the difference between the wavelengths to be determined if λ_{ave} and δd are known.

Notes on equipment

Equipment list

- Michelson interferometer
- Light sources – mercury and sodium

Equipment guidance

Michelson Interferometer Lever Arm Ratio

The lever arm ratio on the Michelson Interferometer should be 5:1, but it is recommended that this is confirmed before detailed measurements are made.

- Switch on the mercury lamp and clip a metal pointer to the diffusing screen, D. Look into the interferometer you should see three images of the pointer: two from M_1 and one from M_2. Adjust the screws on M_2 until the image from M_2 coincides exactly with the brighter image from M_1. Fine fringes will be made visible, located between infinity and the surface of M_1. These fringes can be made larger and eventually circular by fine adjustment of the screws at M_2. Continue to adjust M_2 until the circular fringe pattern is centred in the circular viewing aperture. It may be necessary to move M_1 by means of the micrometer so that several circular fringes appear in the field of view.

- Using the mercury lamp with the orange filter obtain a fringe pattern and adjust the mirrors so that a pattern of reasonably spaced and contrasted circular fringes is obtained. As the micrometer is adjusted – slowly – there will be fluctuations in the clarity of the fringe pattern. There should be a point where the fringe pattern disappears entirely. Note the reading on the micrometer at this point.

- Repeat this process for a number of different vanishing point positions. You can now calculate an average value for the movement of the micrometer needed to move between successive vanishing points, $(\delta D)_{\text{ave}}$. The best way to do that is by plotting δD against vanishing point numbers – the gradient will give you the average.

- The wavelengths of the mercury orange doublet are

$$\boxed{\lambda_1 = 579.07\ \text{nm and } \lambda_2 = 576.96\ \text{nm}}$$

so [5] will give you the distance that M_1 has moved, δd. [6] will then allow you to work out the value of the lever arm reduction ratio, α:

$$\alpha = \frac{distance\ travelled\ by\ micrometer}{distance\ travelled\ by\ mirror\ M_1} = \frac{\delta D}{\delta d} \qquad [6]$$

Counting fringes to measure average wavelength, λ_{ave}

- Counting fringes in the sodium interference patterns can take some practice. To start with, you should adjust the micrometer until there is a circular fringe just about to appear at the centre of the pattern and take a note of the reading on the micrometer.

- You want to record the distance you have moved the micrometer for a certain number of fringes. Taking a measurement for every single fringe would be extremely difficult but keeping track of exactly how many you have counted gets harder the more you do. A good balance would be to count 20 fringes, note the position of the micrometer; count another 20 fringes and note the new position; and so on. You should aim to get at least to 120 if possible, though you may find the pattern fades too much to allow this due to the issues discussed earlier about using a source with two close wavelengths.

- A vertical line can be drawn on the diffuser so that it appears in the field of view to the right but close to the centre of the ring pattern. The fringes emerging from (or collapsing to) the centre can be more easily counted as they cross this vertical line.

- From [2] we can write

$$2\delta d = m\lambda_{ave} \Rightarrow \delta d = m\frac{\lambda_{ave}}{2} = \frac{\delta D}{\alpha}$$

$$\boxed{\Rightarrow \delta D = \frac{\alpha\lambda_{ave}}{2}m} \qquad [7]$$

Hence a $\delta D - m$ plot will have a gradient that will yield λ_{ave}.

Determining separation of sodium lines

- The method used to determine the level arm ratio can be used here, but using the sodium light for which λ_{ave} has ben obtained. Equation [6]

$$(\lambda_1 - \lambda_2) = \frac{\lambda_{ave}^2}{2\delta d}$$

Now since we already know α we can convert from δD to δd and determine $(\lambda_1 - \lambda_2)$.

Original script: Peter Law

Updated script: Peter H Sneddon