Advanced Higher Physics: Assignment Support

Astronomy & Physics Education Group

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Measurement of refractive index of glass by Michelson Interferometer

Introduction

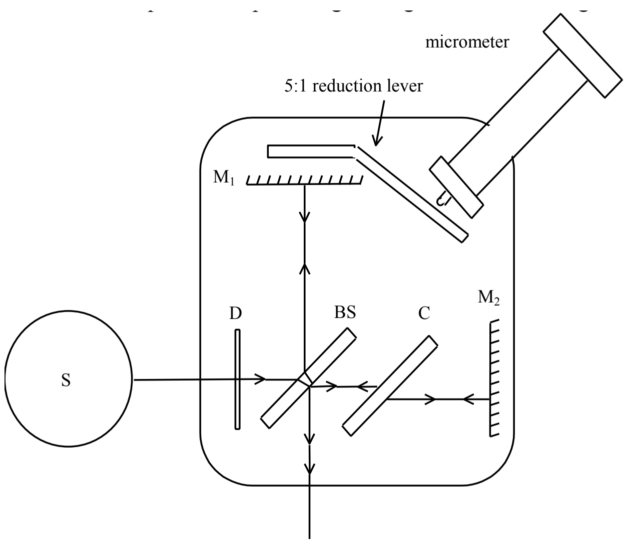

Figure 1: Schematic diagram of a Michelson interferometer.

A Michelson Interferometer creates interference patterns by taking a single source of light, splitting that light into two different paths and then recombining them. A schematic diagram showing the standard form of a Michelson Interferometer is shown in Figure 1. S is the source of the light, which is diffused by screen D and then split into two parts by partial reflection at the beam splitter, BS. The resulting two light beams then follow different paths (vertically and horizontally in the Figure) and, after reflection at M_1 and M_2, are brought together again to produce interference fringes. The compensator plate, C, is used to equalise the path lengths in glass of the two light beams.

In this particular make of interferometer, movement of mirror M_1 is controlled via a level arm, which has a reduction ratio of 5:1 – i.e. the lever makes the mirror M_1 move only a fifth of the distance moved by the micrometer tip. The value of the reduction ratio is approximate – determination of the exact ratio is need before detailed experiments can be carried out.

Circular fringes are produced using monochromatic light when the mirrors M_1 and M_2 are exactly perpendicular. The formation of these fringes may be more readily understood by considering Figure 2 which illustrates the essential features of the interferometer. Here the interferometer has been "unfolded". In this figure, M_2 has been replaced by M'_2, the reflection of M_2 in the beam splitter, as seen by the observer.

Due to reflection in the real interferometer the source S appears to be behind the observer who sees two virtual images of S, S_1 and S_2 reflected by M_1 and M'_2. If M_1 is parallel to M'_2 and if the separation of M_1 and M'_2 is d, then the distance S_1 to S_2 will be $2d$.

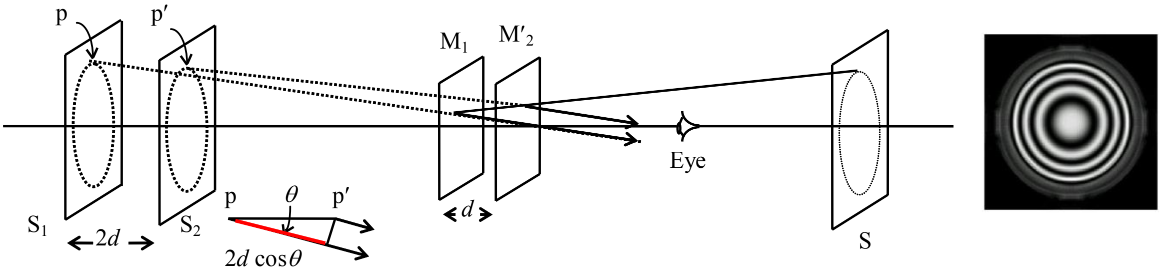

Figure 2:

If the incident light has wavelength λ, then for $2d = m\lambda$ (where m is an integer) all light reflected normal to the mirrors will be in phase. However, rays reflected at an angle θ will not, in general, be in phase.

The path differences from rays coming to the eye (focused to receive parallel light) from corresponding points p and p' (as shown in red in Figure 2) is $2d\cos(\theta)$. Thus the rays will reinforce and produce a maximum (assuming there are no phase changes introduced by the reflections of the light beams) when

$$2d\cos(\theta) = m\lambda$$

[1]

For parallel mirrors, the set up is cylindrically symmetric. When you look at the interferometer, the rays will be at a slight angle to your eye, therefore you will see circles. If M_1 is slowly moved inwards, those circular fringes collapse inwards and vanish at the centre axis, where $\cos(\theta) = 1$ and hence [1] simplifies to

$$2d = m\lambda$$

[2]

A circular fringe will disappear each time $2d$ decreases by λ, or equivalently when d decreases by $\frac{\lambda}{2}$. Conversely, if M_1 is moved outwards, therefore increasing the distance d, the circular fringes will emerge from the centre and expand, a circular fringe emerging each time $2d$ increases by λ, or equivalently when d increases by $\frac{\lambda}{2}$.

Using monochromatic light, the circular fringes are visible for very long path differences in the interferometer. If the source consists of two closely spaced lines of wavelengths λ_1 and λ_2, where $\lambda_1 > \lambda_2$ (such as when using the Sodium Doublet) the “visibility” of the fringe pattern fluctuates regularly through highly contrasted maxima and vanishings of the fringe pattern as the path difference is changed. This fluctuation is caused by the two fringe patterns corresponding to the two wavelengths getting in and out of step as d varies. With circular fringes the path difference can be adjusted until the pattern appears to vanish, at least near the centre of the pattern. This first occurs when the fringe numbers for the two wavelengths differ by $\frac{1}{2}$ because then the centre of the fringe pattern for one wavelength is light when that for the other is dark, leading to minimum visibility. If this first vanishing of the fringe pattern occurs when the mirror separation is d then the fringe numbers are $\frac{2d}{\lambda_1}$ and $\frac{2d}{\lambda_2}$ respectively and so

$$\Delta m = \frac{2d}{\lambda_2} - \frac{2d}{\lambda_1} = \frac{1}{2}$$

[3]

If the separation of the mirrors is now increased by an amount δd until the next vanishing of the fringe pattern is obtained then this will occur when the difference in fringe number becomes 3/2 and so ...

$$\Delta m = \frac{2(d+\delta d)}{\lambda_2} - \frac{2d(d+\delta d)}{\lambda_1} = \frac{3}{2}$$

[4]

[4]-[3] then gives us ...

$$\frac{2\delta d}{\lambda_2} - \frac{2\delta d}{\lambda_1} = 1$$

$$\Rightarrow \ \delta d = \frac{\lambda_1 \lambda_2}{2(\lambda_1 - \lambda_2)}$$

[5]

If a doublet of known wavelengths, e.g. the mercury orange doublet, is used then δd can be determined using [5].

If the change in the micrometer reading δD is recorded, and δd obtained as described then the true level reduction ratio, α, can be obtained as follows ...

$$\alpha = \frac{distance\ travelled\ by\ micrometer}{distance\ travelled\ by\ mirror\ M_1} = \frac{\delta D}{\delta d}$$

[6]

Since white light fringes are obtained when the two optical paths are identical, introducing a glass plate into one pf the paths will immediately destroy the white light fringe pattern as it will effectively lengthen that path.

If an optically flat plate of glass of thickness t and refractive index n is introduced into the path from the beam splitter BS to mirror M_1 and aligned parallel to M_1 the length of the path between M_1 and BS in air will be reduced by an amount t.

But the light has to pass through a thickness t of glass of refractive index n and so this will increase the optical path by an amount nt. This is because there are n more waves per unit length in the glass since the speed of light in the glass is $\frac{1}{n}$th of what it is in air. Hence the overall change in the optical path length from the beam splitter to mirror M_1 will be an increase of amount $nt - t = t(n-1)$.

The white light fringe pattern will not be restored until that optical path is shortened to equal again the other optical path.

If the distance that the mirror M_1 must be moved towards the beam splitter to restore the white light fringe pattern is δd then

$$\delta d = t(n - 1)$$

i.e.

$$n = \frac{\delta d}{t} + 1$$

[7]

If d and t are measured, [7] above can be used to determine the refractive index n of the glass.

Notes on equipment

Equipment list

- Michelson interferometer
- Light sources – mercury and white light

Equipment guidance

Michelson Interferometer Lever Arm Ratio

The lever arm ratio on the Michelson Interferometer should be 5:1, but it is recommended that this is confirmed before detailed measurements are made.

- Switch on the mercury lamp and clip a metal pointer to the diffusing screen, D. Look into the interferometer you should see three images of the pointer: two from M_1 and one from M_2. Adjust the screws on M_2 until the image from M_2 coincides exactly with the brighter image from M_1. Fine fringes will be made visible, located between infinity and the surface of M_1. These fringes can be made larger and eventually circular by fine adjustment of the screws at M_2.

Continue to adjust M_2 until the circular fringe pattern is centred in the circular viewing aperture. It may be necessary to move M_1 by means of the micrometer so that several circular fringes appear in the field of view.

- Using the mercury lamp with the orange filter obtain a fringe pattern and adjust the mirrors so that a pattern of reasonably spaced and contrasted circular fringes is obtained. As the micrometer is adjusted – slowly – there will be fluctuations in the clarity of the fringe pattern. There should be a point where the fringe pattern disappears entirely. Note the reading on the micrometer at this point.

- Repeat this process for a number of different vanishing point positions. You can now calculate an average value for the movement of the micrometer needed to move between successive vanishing points, $(\delta D)_{\mathrm{ave}}$. The best way to do that is by plotting δD against vanishing point numbers – the gradient will give you the average.

- The wavelengths of the mercury orange doublet are

$$\boxed{\lambda_1 = 579.07\ \mathrm{nm}\ \text{and}\ \lambda_2 = 576.96\ \mathrm{nm}}$$

so [5] will give you the distance that M_1 has moved, δd. [6] will then allow you to work out the value of the lever arm reduction ratio, α:

$$\alpha = \frac{distance\ travelled\ by\ micrometer}{distance\ travelled\ by\ mirror\ M_1} = \frac{\delta D}{\delta d} \qquad [6]$$

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