Advanced Higher Physics: Assignment Support

Astronomy & Physics Education Group

School of Physics & Astronomy

University of Glasgow

Determination of g by Owen's bar pendulum

A compound pendulum

Any rigid body suspended in the vertical plane on a horizontal axis through a point other than its centre of mass, and free to oscillate, constitutes a compound pendulum. Figure 1 shows such a rigid body, of mass m, suspended from point O which is a distance h from the centre of mass, CM.

When displaced by a small angle θ from its equilibrium, the pendulim experiences a restoring torque

$$\tau = -mgh\sin(\theta) \approx -mgh\theta$$

if θ is very small. This restoring torque produces an angular acceleration

$$\alpha = \frac{d^2\theta}{dt^2}$$

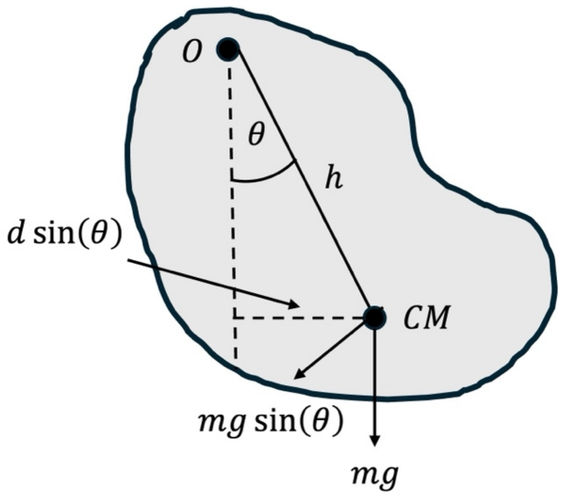

Figure 1: A compound pendulum

Newton's 2nd law for rotating masses states that the net restoring torques must equal the product of the moment of inertia, I, and the angular acceleration, α. We can therefore write:

$$I\alpha = I\frac{d^2\theta}{dt^2} = -mgh\theta \Rightarrow I\frac{d^2\theta}{dt^2} + mgh\theta = 0$$

[1]

I can be expressed in terms of the radius of gyration, k, of the pendulum about its centre of mass and the distance, h:

$$I = m(k^2 + h^2)$$

[2]

Substituting [2] into [1] gives ...

$$m(k^2 + h^2)\frac{d^2\theta}{dt^2} + mgh\theta = 0$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{gh}{(k^2 + h^2)}\theta = 0$$

[3]

Comparing this with the standard equation for Simple Harmonic Motion $\frac{d^2y}{dt^2} + \omega^2 y = 0$ where we are using θ as our displacement variable, we can see that and we see that

$$\omega = \sqrt{\frac{gh}{(k^2 + h^2)}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{(k^2 + h^2)}{gh}}$$

[4]

Plotting T against h will produce a curve with a minimum at the point where $h = k$. Figure 2 illustrates an example of this where $k = 0.05$ m – the $T - h$ plot minimises at $h = 0.05$ m.

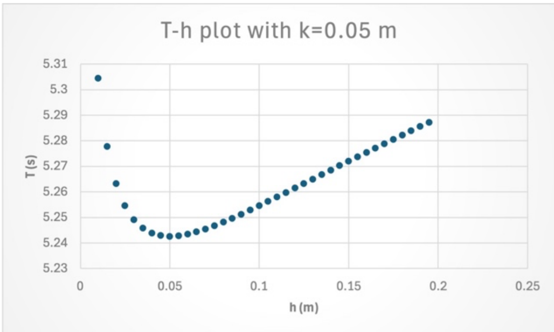

Figure 2: T-h plot

At the point where $h = k$ [4] provides an expression for the minimum period, $T_{\min}$, and from that g can be found

$$T_{\min} = 2\pi\sqrt{\frac{(k^2 + k^2)}{gk}} = 2\pi\sqrt{\frac{2k}{g}}$$

$$\boxed{\Rightarrow g = \frac{8\pi^2 k}{T_{min}^2}}$$

[5]

The Owen's Bar Pendulum

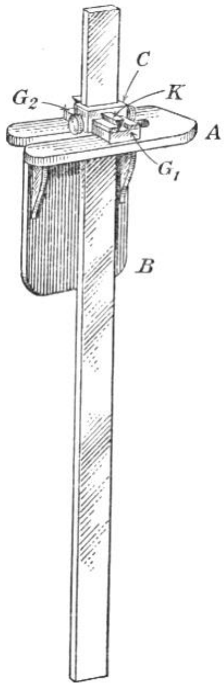

Figure 3

The Owen's Bar pendulum is a specific example of a compound pendulum which has a simple geometry – a uniform rectangular bar. This is held within a carriage that contains knife edges, which allows the bar to swing. The position of the carriage can be altered. Figure 3 is an illustration of the original. As it is a uniform rectangular bar, it is possible to determine its radius of gyration about a perpendicular axis through its centre of mass by simple measurement of its breadth (a) and length (b). If it is then suspended from a point a distance from its centre of mass equal to its radius of gyration, and its period of oscillation through small oscillations measured, then g can be determined by a simple equation.

In Figure 3, C is the holder for the bar, carrying the knife edges K. These rested on glass plates G placed on the support A clamped to the bench. The pendulum in the lab is similar but dispenses with the glass plates.

Figure 4 shows a simplified model of the pendulum, aligned such that the bar is sitting in the $x-y$ plane. If the bar has mass M, then the moment of inertia about an axis through its centre of mass, aligned perpendicular to the bar (i.e. along the z-axis), can be calculated using the perpendicular axes theorem.

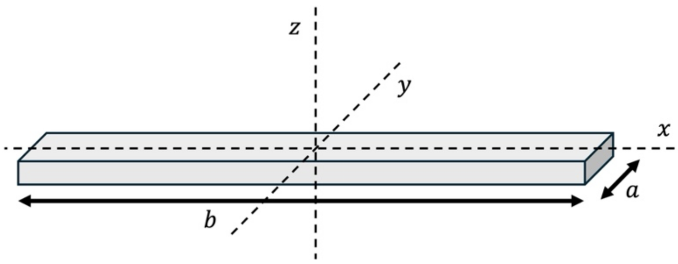

Figure 4: Owen's Bar Schematic

$$I_z = I_x + I_y$$

For this geometry, $I_x = \frac{1}{12}Ma^2$ and $I_y = \frac{1}{12}Mb^2 \Rightarrow I_z = \frac{1}{12}M(a^2+b^2)$

Comparing this with [2] we see that $k^2 = \frac{1}{12}(a^2+b^2)$, and so

$$\Rightarrow I_z = Mk^2$$

If the lengths a and b are measured, then k can be obtained. If Owen's bar pendulum is suspended from a point a distance k from its centre of mass, then $h = k$, so

$$T = 2\pi\sqrt{\frac{2k}{g}} \Rightarrow g = \frac{8\pi^2 k}{T^2}$$

[5*]

An Owen's Bar pendulum can, therefore, be used in two ways to determine the value of g. If the centre of mass is identified, then the bar can be suspended about axes at various distances h from that centre and the corresponding periods measured. Plotting the results would allow the point where $h = k$ to be found, and then [5] would yield g.

Alternatively, the dimensions of the bar can be measured, allowing k to be determined. This allows the bar to be suspended a distance $h = k$ from the centre of mass; once the period is then obtained g can be calculated.

Notes on equipment

Equipment list

The equipment provided for this Experiment are:

- The Owen's Pendulum
- Support carriage for suspending the pendulum
- Knife-edge block
- A digitimer and lightgate for measuring period
- Vernier callipers

Equipment guidance

Digitimer and lightgate:

Figure 5 shows the digitimer.

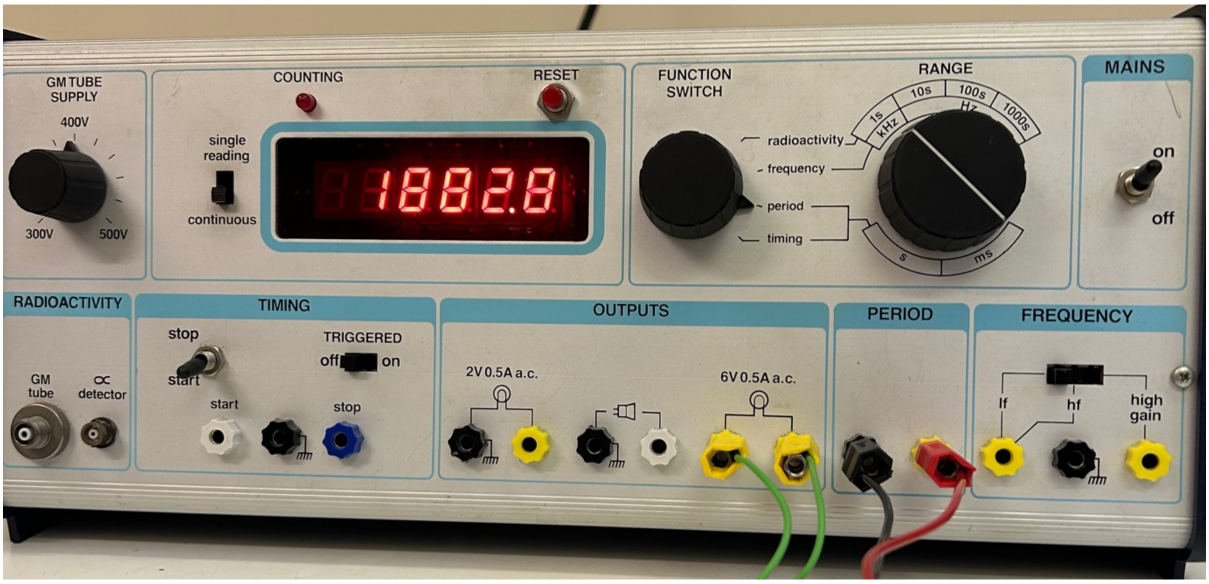

Figure 5: Digitimer

- To make sure it is working correctly, the FUNCTION SWITCH must be set to “period” and the RANGE to “ms”. It must also be set to take “continuous” readings.
- To make sure the period is correctly recorded, the lightgate must be positioned such that small oscillations of the pendulum allow the end of the rod to interrupt, but not pass completely through, the lightgate beam.

Knife edge block:

- Take care when balancing the bar on this – never take your hands too far from the bar as it falls easily. Be careful where you set it up.

Original script: Peter Law

Updated script: Peter H Sneddon