Advanced Higher Physics: Assignment Support

Astronomy & Physics Education Group

School of Physics & Astronomy

University of Glasgow

Determination of g by Kater pendulum

Simple and compound pendulums

The reversible, or Kater, pendulum was devised by Henry Kater in 1817 to be used for determining the local value of g. Its advantage lay in the fact that the position of the centre of mass did not need to be accurately determined. It is an example of a *compound pendulum*. A simple pendulum is an idealised model, where there is a point-mass suspended at the end of a massless, unstretchable string. If the mass is pulled through a small angle θ and then released, the mass will oscillate about its equilibrium position with angular frequency ω and period T given by the following expressions:

$$\omega = \sqrt{\frac{g}{L}}; T = 2\pi\sqrt{\frac{L}{g}}$$

[1a,b]

where L is the length of the pendulum string and g is the acceleration due to gravity. A compound pendulum better models reality, where the mass of the oscillating object is distributed, rather than concentrated in a single point.

Consider the irregularly shaped object shown in Figure 1. This can turn about an axis through the point O. In equilibrium, its centre of mass (CM) is directly below this point. In the figure this point has been moved through and angle θ. If we define the distance from O to CM to be h, and say that the total mass is m, and that the masses moment of inertia about O is I, then the weight of the object will create a restoring torque:

$$\tau = -mgh\sin(\theta)$$

If we assume that the displacement angle is small., this simplifies to

$$\tau = -mgh\theta$$

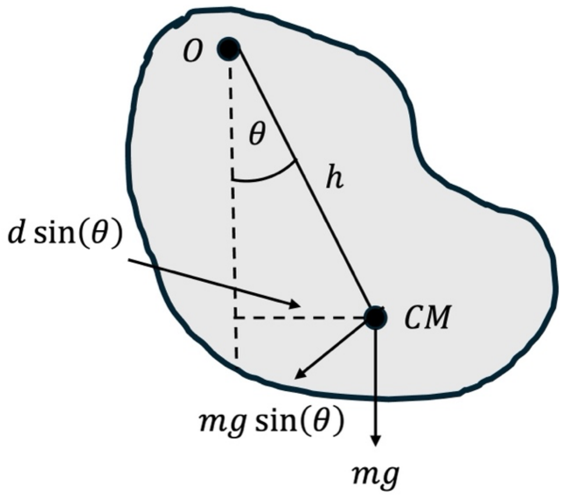

Figure 1: A compound pendulum

[2]

The minus sign is there as this is a restoring torque. Using the angular version of Newton's laws – and assuming there are no other forces in play – we can state:

$$-mgh\theta = I\frac{d^2\theta}{dt^2}$$

$$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{mgh}{I}\theta = 0$$

[3]

Consider the units of these terms: m is measured in kg; g in ms^{-2}; h in m; I in kgm^2. This means that the units of $\frac{mgh}{I}$ are $\frac{\mathrm{kgms}^{-2}\mathrm{m}}{\mathrm{kgm}^2} = \frac{\mathrm{kgm}^2\mathrm{s}^{-2}}{\mathrm{kgm}^2} = \mathrm{s}^{-2}$. Therefore we can see that here the angular frequency is given by the square root of this combination:

$$\omega = \sqrt{\frac{mgh}{I}}$$

and in turn,

$$T = 2\pi\sqrt{\frac{I}{mgh}}$$

[4a,b]

For a compound pendulum, I can be expressed in terms of h and k, where k is the radius of gyration – this is the distance from O that a point mass of mass equal to the distributed mass would be located to generate the same motion.

$$I = m(h^2 + k^2)$$

$$\Rightarrow T = 2\pi\sqrt{\frac{m(h^2 + k^2)}{mgh}} = 2\pi\sqrt{\frac{h^2 + k^2}{gh}}$$

[5]

As in the case of the simple pendulum, the mass plays no part in the period calculation.

The Kater Pendulum

There are various designs of the Kater pendulum. The design of the one that will be used at the University of Glasgow is illustrated in Figure 2.

The pendulum consists of a long rod, to which two masses have been attached. At one end there is a large, fixed mass (M), below which are a fixed set of knife edges, labelled K_1 which the pendulum balances on. Further down is a second mass (m) which can be moved along the rod. Its position can be fixed by a locking screw L. There are a second set of knife edges, K_2, identical to those at K_1, but pointing in the opposite direction. This allows the pendulum to be suspended with the large mass M uppermost using set K_1, or at the lower end of the pendulum, using set K_2.

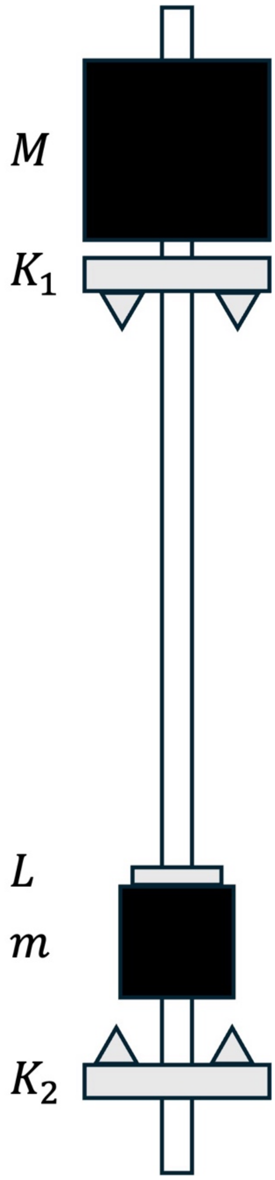

Figure 2: Kater pendulum

When the pendulum is suspended from knife edges K_1, the period of oscillation, T_1, is given by the compound pendulum equation:

$$T_1 = 2\pi\sqrt{\frac{h_1^2 + k^2}{gh_1}}$$

[6]

Here h_1 is the distance from the knife edges K_1 to the centre of mass of the pendulum, and k is the radius of gyration of the pendulum. Similarly, when the pendulum is suspended from K_2, the period of oscillation T_2 is given by:

$$T_2 = 2\pi\sqrt{\frac{h_2^2 + k^2}{gh_2}}$$

[7]

where h_2 is the distance from the knife edges K_2 to the centre of mass of the pendulum.

Rearranging [6] and [7] gives us ...

$$T_1 = 2\pi\sqrt{\frac{h_1^2 + k^2}{gh_1}} \Rightarrow \frac{gh_1T_1^2}{4\pi^2} = h_1^2 + k^2$$

[8]

$$T_2 = 2\pi\sqrt{\frac{h_2^2 + k^2}{gh_2}} \Rightarrow \frac{gh_2T_2^2}{4\pi^2} = h_2^2 + k^2$$

[9]

then subtracting [9] from [8] eliminates k:

$$\frac{gh_1T_1^2}{4\pi^2} - \frac{gh_2T_2^2}{4\pi^2} = h_1^2 - h_2^2 \Rightarrow \frac{g}{4\pi^2}(h_1T_1^2 - h_2T_2^2) = h_1^2 - h_2^2$$

$$\Rightarrow \frac{4\pi^2}{g} = \frac{h_1T_1^2 - h_2T_2^2}{h_1^2 - h_2^2}$$

[10]

In 1826 the German mathematician and astronomer Friedrich Bessel showed that [10] could be written in the following form:

$$\frac{4\pi^2}{g} = \frac{T_1^2 + T_2^2}{2(h_1 + h_2)} + \frac{T_1^2 - T_2^2}{2(h_1 - h_2)}$$

[11]

Provided that $M \gg m$ then $h_1 - h_2$ is going to be a relatively large value. Further, if adjustments are made so that T_1 and T_2 are brought as close together as possible, such that $T_1^2 - T_2^2$ becomes very small, then we can ignore the 2nd term above and simply write:

$$\frac{4\pi^2}{g} = \frac{T_1^2 + T_2^2}{2(h_1 + h_2)}$$

$$\Rightarrow \frac{g}{4\pi^2} = \frac{2(h_1 + h_2)}{T_1^2 + T_2^2}$$

$$\boxed{\Rightarrow g = \frac{8\pi^2(h_1 + h_2)}{T_1^2 + T_2^2}}$$

[12]

Notes on equipment

Equipment list

he equipment provided for this Experiment are:

- The Kater Pendulum
- Support bracket for suspending the pendulum
- Allen key for loosening and tightening locking screw
- A digitimer and lightgate for measuring period
- Kevlar safety gloves
- Metre rule

Equipment guidance

Pendulum:

- The utmost care must be taken when working with the Kater pendulum as the knife edges that it balances on are extremely sharp. Always wear Kevlar Safety Gloves when setting the pendulum to balance on K_1 and K_2.

Digitimer and lightgate:

Figure 3 shows the digitimer.

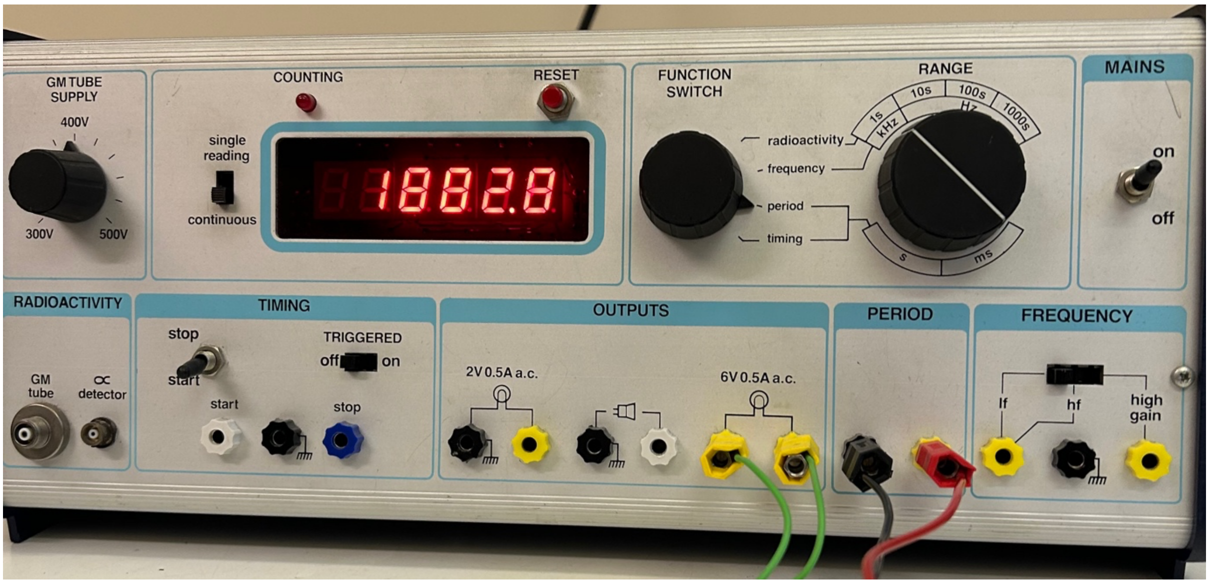

Figure 3: Digitimer

- To make sure it is working correctly, the FUNCTION SWITCH must be set to "period" and the RANGE to "ms". It must also be set to take "continuous" readings.
- To make sure the period is correctly recorded, the lightgate must be positioned such that small oscillations of the pendulum allow the end of the rod to interrupt, but not pass completely through, the lightgate beam.

Original script: Peter Law

Updated script: Peter H Sneddon