Advanced Higher Physics: Assignment Support

Astronomy & Physics Education Group

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Determination of g by Bifilar suspension

Introduction

In this experiment a bar is suspended horizontally by two thin wires of equal length and made to oscillate, through a small angle, in the horizontal plane. Such an arrangement is called a bifilar suspension. The simplified arrangement of a bifilar suspension is shown in Figure 1.

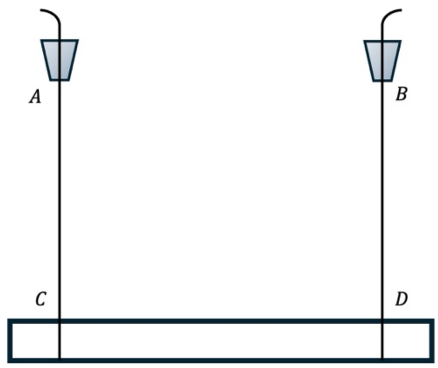
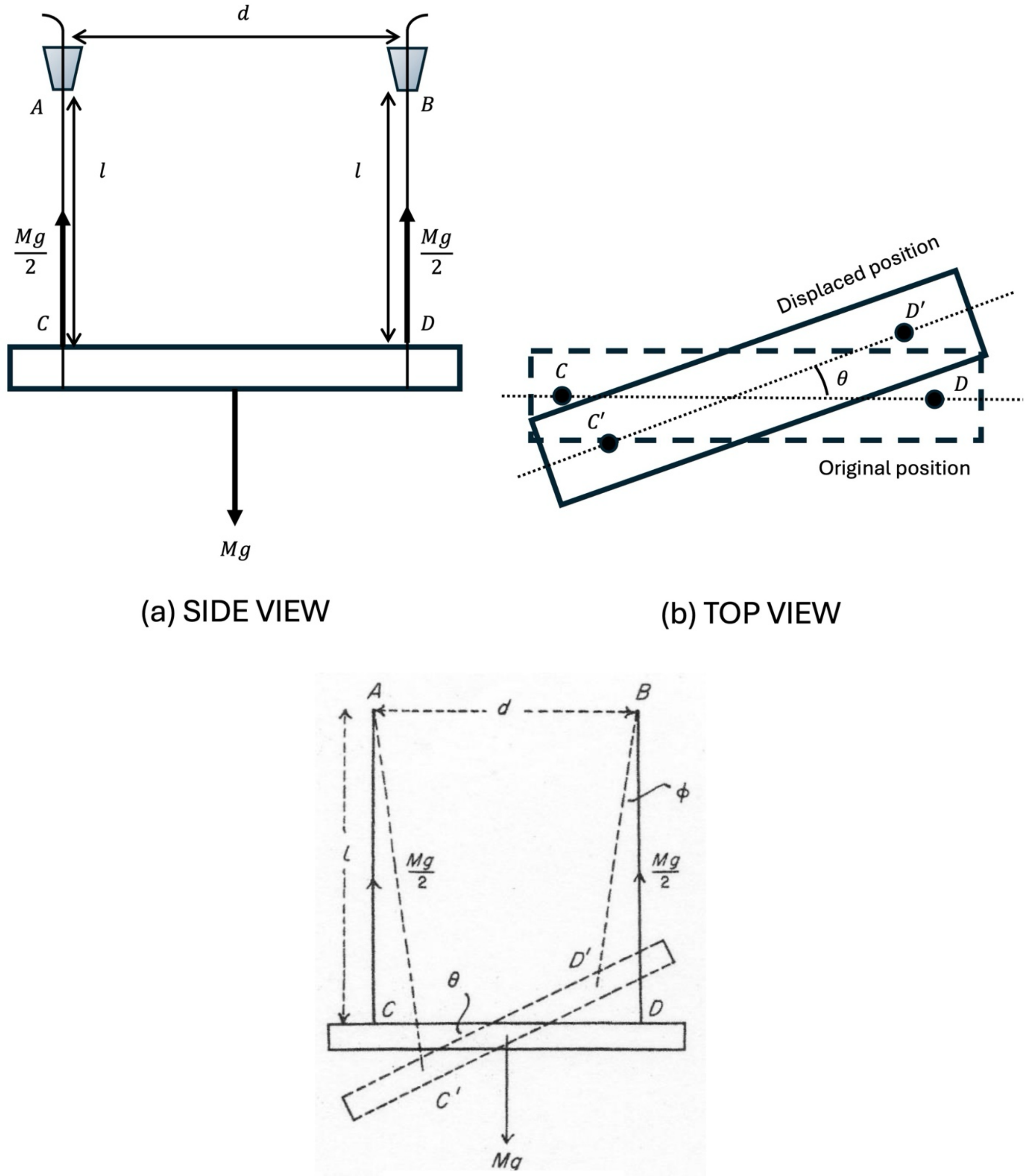


Figure 1: A bifilar pendulum

AC and BD represent the wires suspending the bar. When it is given a small angular displacement, θ, about a vertical axis through the centre it will oscillate in the horizontal plane.

A more detailed diagram of the experimental set up is shown in Figure 2. The set up is tricky to illustrate in two dimensions but the combination of 2a, 2b and 2c gives an overview of what is happening. Here we have defined the lengths of the supporting wires to be l and the separation of those wires to be d. If the mass of the suspended bar is M then the weight of the bar will be Mg, and the tension in each string will be $\frac{Mg}{2}$.



(c) PROJECTION

Figure 2: A bifilar pendulum rotated

When the bar is turned in the horizontal plane through an angle of θ, the points where the wires contact the bar also move in the horizontal plane. This also causes the wires to change orientation with respect to their original vertical alignment through an angle ϕ. If we call the original points of contact C and D, and the rotated points of contact C' and D', then we can right

$$CC' = \tfrac{d}{2}\theta \text{ and } DD' = l\phi$$

Here we are assuming that our angles are small, which also allows us to state that the arc length CC' and arc length DD' are equal and so we can write

$$l\phi = \frac{d}{2}\theta$$
$$\Rightarrow \phi = \frac{d}{2l}\theta$$

[1]

When the bar is displaced, the tension F in each wire produces a horizontal force; these act to oppose the displacement and restore the bar to its equilibrium position.
The horizontal components of the tension in these wires are ...

$F_{AC} = F\sin\phi$ and $F_{BD} = F\sin\phi$ and since ϕ is very small, $F_{AC} = F_{BD} = F\phi$.

Acting together, these components produce a restoring torque Γ about the centre of the bar, where

$$\Gamma = -2F\phi\frac{d}{2} = -F\phi d = -Fd\frac{d}{2l}\theta = -\frac{Fd^2}{2l}\theta$$
$$\Rightarrow \Gamma = -\frac{Mgd^2}{4l}\theta$$

since tension F in each wire is half of the weight.

Our rotational equation of motion for the bar is then ...

$$I\frac{d^2\theta}{dt^2} = -\frac{Mgd^2}{4l}\theta$$

where I is the moment of inertia of the bar.

$$\Rightarrow I\frac{d^2\theta}{dt^2} + \frac{Mgd^2}{4l}\theta = 0$$

[2]

This equation describes simple harmonic motion. And if we compare it to the standard form for SHM, we can see that the period of this oscillation, T, is

$$T = 2\pi\sqrt{\frac{4Il}{Mgd^2}}$$

[3]

Since l, M and d are easily measured, then by measuring the period of oscillation T we can find a value for g provided we can determine the moment of inertia, I, of the bar about its centre.

Figure 3 shows the suspended bar, aligned such that the bar is sitting in the $x - y$ plane. If the bar has mass M, then the moment of inertia about an axis through its centre of mass, aligned perpendicular to the bar (i.e. along the z-axis), can be calculated using the perpendicular axes theorem.

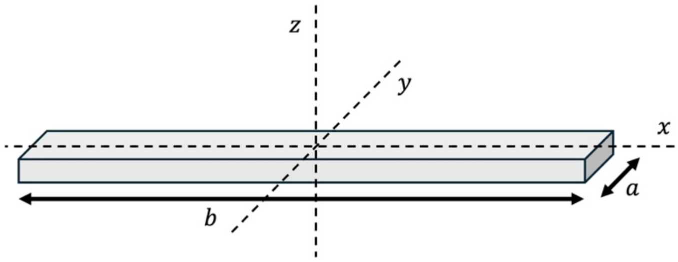


Figure 3: Schematic of suspended

$$I_z = I_x + I_y$$

For this geometry, $I_x = \frac{1}{12}Ma^2$ and $I_y = \frac{1}{12}Mb^2 \Rightarrow I_z = \frac{1}{12}M(a^2 + b^2)$

Comparing this with [2] we see that $k^2 = \frac{1}{12}(a^2 + b^2)$, and so

$$\Rightarrow I_z = Mk^2$$

So by measuring the dimensions of the bar, we can determine the moment of inertia we need.

$$T = 2\pi\sqrt{\frac{4Il}{Mgd^2}} = 2\pi\sqrt{\frac{4Mk^2l}{Mgd^2}} = 2\pi\sqrt{\frac{4k^2l}{gd^2}}$$

$$\boxed{\Rightarrow T = 2\pi\sqrt{\frac{4k^2}{gd^2}} \times \sqrt{l}}$$

[4]

Plotting T against $\sqrt{l}$ will then allow g to be determined.

Notes on equipment

Equipment list

- Bifilar set up
- A digitimer and lightgate for measuring period
- Metre rule
- Vernier callipers

Equipment guidance

Bifilar suspension:

Best to start with longest possible value of l, and don't go below $300\ \mathrm{mm}$

Need to make sure bar is horizontal – spirit level can help here. If required, you can loosen the terminal block screws and adjust the lengths of the wires until the bubble is centred.

Digitimer and lightgate:

Figure 4 shows the digitimer.

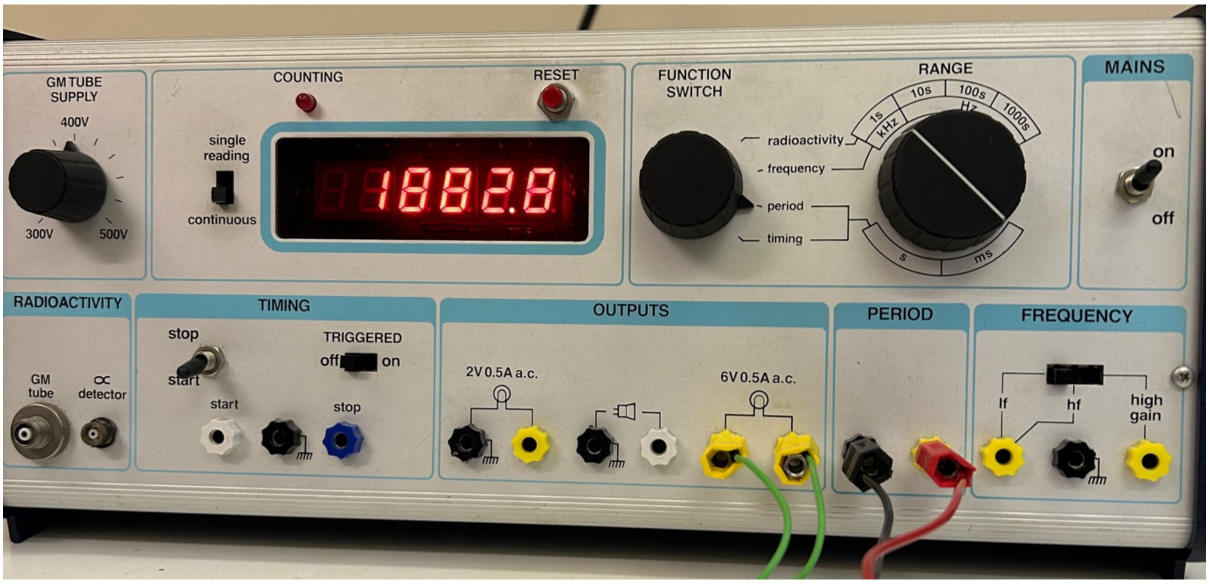


Figure 4: Digitimer

- To make sure it is working correctly, the FUNCTION SWITCH must be set to "period" and the RANGE to "ms". It must also be set to take "continuous" readings.

- To make sure the period is correctly recorded, the lightgate must be positioned such that small oscillations of the pendulum allow the end of the rod to interrupt, but not pass completely through, the lightgate beam.

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