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WORKING PAPER SERIES

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Paper No. 2024-11
October 2024

Equity versus Efficiency: Optimal Monetary and Fiscal Policy in a HANK Economy*

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October 14, 2024

Abstract

We analyze optimal monetary and fiscal policy in a tractable heterogeneous agent New Keynesian (HANK) economy where overlapping generations of households wish to save for retirement and precautionary reasons. Fiscal policy matters most. A Ramsey policy maker faces trade-offs between intra- and inter-generational equity and between equity and efficiency. Intergenerational equity requires the government to issue debt to facilitate saving for retirement, but this drives up interest rates and inhibits household borrowing to mitigate the impact of idiosyncratic shocks. Issuing debt also reduces efficiency since taxes are distortionary. These trade-offs are resolved in favor of equity over efficiency.

Key Words: Heterogeneous Agents Models, Monetary Policy, Fiscal Policy, Inequality
JEL Reference Number: E21, E30, E61, E62, E63

*Vasileios Karaferis is grateful to the ESRC for providing financial support through a PhD scholarship.

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1 Introduction

There is growing interest in issues of inequality in macroeconomics. Politicians have awoken to the possibility that the policy consensus has not always been felt to benefit all of society, leading populist politicians to highlight dissatisfaction with the status quo (Rodrick, 2018). Central banks are also increasingly conscious of the distributional impacts of their policies.[1]

Popular treatments debate the relative importance of intragenerational (Bristow, 2019) versus intergenerational (Sternberg, 2019) inequality. However, technical issues often make it difficult to provide well-founded normative policy recommendations which address these concerns. This paper seeks to make progress in this area by considering (non-trivial) fiscal policies, including the evolution of long-term government debt, alongside monetary policy in an environment where there are meaningful trade-offs between efficiency and equity, both within and across generations.

Specifically, we analyze jointly optimal monetary and fiscal policy in a heterogeneous-agent New Keynesian (HANK) economy. To achieve this, we build upon the insights of Acharya, Challe, and Dogra (2023) and Acharya and Dogra (2020), who develop a tractable heterogenous-agent economy for analyzing optimal monetary policy and monetary policy 'puzzles', respectively. We extend the overlapping-generation model of Acharya et al. (2023) by developing the fiscal side, which was kept deliberately simple given their primary focus on monetary policy.

Specifically, we introduce long-term government debt financed by distortionary labor income taxes, and allow it to evolve over time in accordance with optimal policy. The presence of government debt, coupled with the absence of fiscal transfers to new-born generations, implies that intergenerational inequality is driven not only by idiosyncratic labor supply shocks but also by differing levels of accumulated wealth, which optimal policy must address. By assuming that the disutility of labor supply rises with age, we mimic a desire to save for retirement, which augments the motive for precautionary savings and leads, in equilibrium, to the government optimally issuing plausible levels of government debt to facilitate such saving behavior without sub-optimally suppressing interest rates. The differences in wealth across and within generations endogenously generate differences in exposure to aggregate shocks, which optimal policy will account for.

In Acharya et al. (2023), there are two channels through which monetary policy impacts inequality. First, the extent to which the variance of idiosyncratic risk is pro- or counter-cyclical allows monetary policy to influence the magnitude of that risk following aggregate shocks – the

[1] US Fed chairman Jerome Powell (2020) used his Jackson Hole speech to stress the desirability of running the economy close to maximum employment in order to spread the benefits of economic growth more widely.

‘income-risk’ channel. Second, by lowering interest rates, monetary policy facilitates households’ self-insurance; it becomes cheaper to buy bonds to smooth consumption and easier to borrow against future income when a relatively loose monetary policy expands the economy, raising future income against which one can borrow – the ‘self-insurance’ channel. They examine the cyclicality of consumption risk to explain how monetary policy should respond to shocks. In our model, distortionary tax rates and the level of debt also impact inequality, and they can be even more significant.

Distortionary labor taxation mitigates the initial impact of an indiosyncratic income shock since part of the lost income would have been taxed anyway. However, anticipated future tax rates also affect the household’s ability to borrow against future post-tax income in order to smooth consumption – expectations of higher future taxes increase the costs of earning income to repay any borrowing undertaken to offset a negative idiosyncratic shock. Additionally, distortionary labor income taxation leads to the usual loss of efficiency by discouraging worker effort.

In our overlapping-generation (OLG) economy, the extent to which the government issues debt to facilitate household saving – both for retirement and as protection against idiosyncratic shocks – affects equilibrium real interest rates, both in response to shocks and in the steady state. While monetary policy only has a transitory impact on real interest rates (for as long as prices are sticky) and, through that, inequality, debt policy can have a permanent influence on inequality. As a benchmark, we define a ‘golden rule’ level of steady-state debt that would align the equilibrium real interest rate with households’ rate of time preference, enabling savings for retirement and precautionary savings. We explore how optimal policy diverges from this benchmark.

A policy maker aiming solely to minimize inequality would not issue sufficient debt to ensure that equilibrium interest rates reached this benchmark. Instead, they would allow interest rates to lie below households’ rate of time preference, implying that there are insufficient assets to enable households to save for retirement, let alone accumulate precautionary savings. This will drive a degree of inter-generational inequality as consumption falls throughout households’ lifetimes. The policy maker is prepared to allow this intergenerational inequality since, by suppressing interest rates, the policy maker facilitates household borrowing in the face of negative idiosyncratic shocks, thereby mitigating intragenerational equity. However, the micro-founded social welfare function not only exhibits a concern for equity but also for efficiency. Taking account of efficiency leads the policy maker to issue even less debt, suppressing interest rates further, as issuing debt crowds out economic activity, especially when taxes are distortionary. Thus, the Ramsey policy maker faces a trade-off between both inter- and intra-generational equity and efficiency, which leads them to issue debt but to a lesser extent than needed to facilitate saving for retirement and achieving the 'golden rule' interest rate.

We quantify where the balance is struck in these trade-offs and find that Ramsey policy comes close to achieving the minimum level of inequality – inequality is only 0.001% higher than its minimal value under the fully-optimal Ramsey policy and 0.007% lower than if the policy maker only cared for efficiency. This can also be seen in the steady-state levels of debt issued by the Ramsey policymaker – 54% of GDP – which is only slightly below the level of 58% that would be chosen by a policymaker seeking to minimize inequality alone. In contrast, caring only for efficiency would lead to far lower debt levels of 31% of GDP. Therefore, optimal policy is dominated by a concern for equity.

We then consider the response to aggregate shocks. Again, we examine how fiscal policy contributes to stabilizing the economy in the face of such shocks. The benchmark 'divine coincidence' result – where interest rates would be cut in response to a positive technology shock without generating deflation – only emerges under special circumstances. In our heterogeneous agent OLG economy featuring phased retirement and idiosyncratic income shocks, these conditions are: (i) no fiscal policy other than (ii) a lump-sum tax financed production subsidy that ensures the steady-state is efficient, and (iii) the policy maker only cares about efficiency, not equity. Relaxing these conditions creates a meaningful policy problem. In the absence of fiscal policy, monetary policy engineers a degree of price level control by following an initial period of deflation with a period of positive inflation. Seeking to manipulate expectations in this way is a common feature of Ramsey policy in the New Keynesian model. However, when reducing inequality becomes part of the policy objective, the monetary policy maker relaxes policy further, thereby facilitating households' ability to borrow to offset negative idiosyncratic income shocks. Introducing fiscal policy and government debt implies a non-trivial distribution of wealth that impacts the evolution of consumption inequality. Now, the policy mix in response to shocks relies on the use of distortionary taxation alongside monetary policy to reduce movements in inflation while simultaneously facilitating households' ability to smooth consumption in the face of idiosyncratic shocks.

An interesting element of our modeled economy is that households can hold longer-term government debt, rather than the single period debt often adopted in the literature. Moreover, optimal policy implies significant variance in the holdings of this debt within and across generations. This affects the redistributional impacts of shocks, especially when they are autocorrelated since changes in the entire path of short-term interest rates create capital gains/losses for holders of these longer-term bonds, a dynamic not present with single-period debt. Under the timelessly optimal policies we consider, the policy maker commits to not attempt to unexpectedly induce such redistributions, but when they occur as the result of shocks, the policymaker's policies will be affected by the extent to which the redistributions affect the evolution of inequality.

Literature Review:

Our work is related to the large literature on Bewley (1977), Huggett (1993) and Aiyagari (1994) economies, where households face uninsurable idiosyncratic risk. As already noted, our approach closely follows that of Acharya and Dogra (2020) and Acharya et al. (2023), utilizing the assumptions of CARA utility and normally distributed idiosyncratic shocks to enable us to derive tractable aggregate relationships and a micro-founded measure of social welfare. Specifically, we extend the framework of Acharya et al. (2023) to allow a meaningful role for fiscal policy, which turns out to have significant implications for both the steady-state trade-off between equity and efficiency and the response to shocks.

The broader Heterogeneous Agent New Keynesian (HANK) literature, which combines household heterogeneity with sticky prices, typically focuses on a positive description of the impact on monetary policy in such economies (see Violante, Violante and Sargent, 2023) for overviews of the key insights gained from the literature, and Kaplan and Violante (2018) for a survey of the HANK literature, more generally). This focus is largely due to the computational complexity of modeling optimal policy in an environment where the state space is infinite. Recently, there has been progress in addressing these computational issues, and authors are beginning to explore normative policy issues, particularly relating to the conduct of monetary policy. For example, see Bhandari et al. (2021) and Le Grand et al. (2022) for analyses of optimal policy, and McKay and Wolf (2022) for a characterization of optimal policy rules. Additionally, Nuño and Thomas (2022) utilize results from continuous-time mathematics to track the wealth distribution over time.

A more common approach in addressing normative issues involves making simplifying assumptions to ensure sufficient tractability to facilitate the analysis of optimal policy. For example, it is common to assume that households cannot borrow and government debt is in zero net supply, which implies that, in equilibrium, households do not hold any assets – the *zero liquidity limit*. This assumption eliminates the ability of households to self-insure through saving and/or borrowing and results in a degenerate wealth distribution, thereby allowing for a tractable analysis of optimal policy. Examples of this approach applied to conventional monetary policy include

Bilbiie (2008), Bilbiie (2024a), Hansen, Lin, and Mano (2020), and Challe (2020). Auclert (2019) and Bilbiie (2024b) extend this consideration to fiscal policy.

Our paper adopts a different set of simplifying assumptions, specifically CARA utility and normally distributed idiosyncratic shocks. This approach implies that the welfare costs of inequality are captured by a single variable, which evolves recursively, making the Ramsey policy problem tractable despite the heterogeneity. The cost in doing so is that the marginal propensity to consume is common across households, rather than varying with income or wealth. Nevertheless, our approach allows for precautionary savings and borrowing in response to idiosyncratic shocks. Furthermore, we extend Acharya et al. (2023) and the literature imposing a zero liquidity limit by allowing government debt to be in non-zero supply and determined endogenously. This implies a non-degenerate wealth distribution, and households within and across generations will face different wealth revaluation effects in the face of aggregate shocks.[2]

Our model relies on an OLG structure as in Blanchard (1985) and Yaari (1965) to prevent the distribution of wealth from becoming non-stationary. Optimal policy in such a framework often focuses on the modified Golden Rule of capital accumulation, which states that the marginal product of capital should equal the rate of growth of the population plus the households' rate of time preference – see the textbook treatment in Blanchard and Fischer (1989). Escolano (1992) obtains the same result by considering optimal policy in an OLG economy with endogenous labor supply and various distortionary taxes. This serves as a useful benchmark in interpreting the results in our OLG heterogeneous agent economy subject to idiosyncratic risk.

Roadmap:

We begin by outlining the details of our economy in the next section. Section 3 discusses our measure of social welfare, highlighting the benchmark 'golden rule' policy, where the policymaker issues sufficient government debt to accommodate households' desire to save for both retirement and precautionary reasons. The section also defines optimal policy. The extent to which optimal policy falls short of this benchmark will highlight the nature of the trade-offs facing the Ramsey policy maker. Section 4 details the calibration, while Section 5 explores the nature of the steady-state with particular emphasis on the trade-off between equity and efficiency under optimal policy. Section 6 then examines the optimal policy response to aggregate shocks. We begin by identifying the conditions under which the 'divine coincidence' emerges in our heterogeneous agent economy

[2] Acharya et al (2022) use a fiscal transfer at the point of birth, and apply a wealth tax to existing households, to ensure all households are *ex ante* identical at time t=0. In a previous version of their paper these assumptions were relaxed to include revaluation effects.

before considering the more significant trade-offs facing the policy maker when these conditions do not apply. Section 7 concludes.

2 The Model

Our model follows that of Acharya et al. (2023), which employs Constant Absolute Risk Aversion (CARA) preferences and normally distributed shocks to individual household labor supply to develop a tractable heterogeneous agent model for the analysis of monetary policy. The model is capable of describing both macroeconomic aggregates and measuring social welfare, accounting for heterogeneity. We undertake the following extensions, which make the modelled economy a tractable framework for examining jointly optimal monetary *and* fiscal policies in the presence of household heterogeneity.

First, we allow for the existence of government debt. This endogenously determines a steady-state distribution of wealth, affecting the optimal response to shocks and implying an additional externality absent in models without government debt. Overlapping generations of households will decide how much to save by purchasing government debt, not internalizing the impact of these decisions on the equilibrium real interest rate – a feature not present in representative agent models and absent in the heterogeneous agent model of Acharya et al. (2023) when government debt is in zero net supply (see Acemoglu, 2008, chapter 9, for a discussion).

Second, in exploring the impact of variation in distortionary tax rates, we do not allow the policy maker access to lump-sum transfers as a policy instrument to finance the government's activities. As a result, raising tax revenues to finance government consumption and service government debt will add to the distortions associated with monopolistic competition. Distortionary taxation will also impact post-tax inequality generated by idiosyncratic income shocks. Moreover, we do not employ a subsidy with which to offset the inefficiencies due to monopolistic competition.

Third, we assume that the disutility of supplying labor income decreases with age in order to mimic economic retirement. This approach generates a desire to save in anticipation of falling incomes, akin to saving for retirement, and ensures our model features a plausible level of government debt under the Ramsey policy.

Fourth, as we wish to consider the Ramsey policy problem for such an economy, we develop a measure of social welfare that accounts for both idiosyncratic shocks within generations and intergenerational inequality driven by the evolution of the wealth distribution over the life cycle. It also captures how these factors interact and affect the welfare implications of aggregate shocks.

2.1 Households

The economy is populated by cohorts of Blanchard-Yaari households that have constant survival probability in any period, $0 < \vartheta < 1$, see Blanchard (1985). At any time t, an individual i who belongs to the generation born at time $s \leq t$ derives utility from real private consumption $c_t^s(i)$ and real government consumption G_t. They also derive disutility from labor supply, $l_t^s(i)$, and, exogenously, disutility rises with age reflecting a desire to retire, $\Theta_t^s = \varkappa(t-s)$. This gradual withdrawal from the labor market will create a desire to save for 'retirement' and will ensure that the government wishes to issue a plausible level of government debt in the Ramsey steady-state. Crucially, households face uninsurable idiosyncratic shocks to disutility from labour $\xi_t^s(i) \sim N(\bar{\xi}, \sigma_t^2)$; these shocks are independent across time and individuals. The variance of this shock may vary with economic activity. There is no aggregate risk.

We assume CARA preferences so utility takes form,

$$U_s = \mathbb{E}_i \sum_{t=s}^{\infty} (\beta\vartheta)^{t-s} \left(-\frac{1}{\gamma} e^{-\gamma(c_t^s(i)+\chi G_t)} - \rho e^{\frac{1}{\rho}(l_t^s(i)+\Theta_t^s-\xi_t^s(i))} \right)$$

Households invest in long and short term nominal actuarial bonds $\mathcal{A}_t^{L,s}(i)$ and $\mathcal{A}_t^{S,s}(i)$. The short-term bonds are issued at price $\tilde{q}_t$, paying out one unit of currency one period later. While, following Woodford (2001), the longer-term bonds, issued at price $\tilde{P}_t^M$, pay an initial coupon of one unit of currency which falls to ϱ^s, s period's later. This geometrically declining coupon is equivalent to a geometric distribution of zero coupon bonds of increasing maturity, such that the implicit average maturity of the bond in a zero inflation environment is, $(1-\varrho\beta)^{-1}$. Longer maturity debt matters as, following shocks, the revaluation effects on wealth held in the form of longer-term bonds through fluctuations in bond prices will be greater, which, in turn, will affect the impact of that shock on the distribution of wealth – see Leeper and Leith, 2016 for a discussion. Households receive after tax-wages, $(1-\tau_t) P_t w_t l_t^s(i)$, where the labor income tax, levied at rate τ_t, is the the sole source of government tax revenues in our benchmark model. We also introduce a lump-sum tax, $P_t T_t$, which will used to replace distortionary taxation as a means of eliminating the effects of tax distortions for expositional purposes only. Each household receives dividends, $P_t d_t$.[3] Their budget constraint at time t is

$$\begin{aligned}&P_t c_t^s(i) + \tilde{P}_t^M \mathcal{A}_{t+1}^{L,s}(i) + \tilde{q}_t \mathcal{A}_{t+1}^{S,s}(i)\\ &= \left(1 + \varrho \tilde{P}_t^M\right) \mathcal{A}_t^{L,s}(i) + \mathcal{A}_t^{S,s}(i)\\ &+ (1-\tau_t) P_t w_t l_t^s(i) + P_t d_t - P_t T_t\end{aligned}$$

Each individual is born with zero bond holdings, $\mathcal{A}_s^{L,s} = \mathcal{A}_s^{S,s} = 0$ and there is no fiscal transfer to newborns and/or wealth tax on existing households to ensure *ex ante* equality between all households as in Acharya et al. (2023).

Define the ratio of the number of each type of assets to the price level as,

$$a_t^{J,s}(i) = \frac{\mathcal{A}_t^{J,s}(i)}{P_{t-1}}, J \in \{L, S\}$$

and introduce a measure of real assets

$$A_t^s(i) = \frac{\left(1 + \varrho \tilde{P}_t^M\right) a_t^{L,s}(i) + a_t^{S,s}(i)}{(1+\pi_t)} \tag{1}$$

Then, we rewrite the budget constraint in real terms,

$$\frac{\vartheta}{R_t} A_{t+1}^s(i) = A_t^s(i) + y_t^s(i) - c_t^s(i) \tag{2}$$

where net household income is defined as,

$$y_t^s(i) = \eta_t l_t^s(i) + d_t - T_t, \tag{3}$$

the post-tax wage is

$$\eta_t = (1-\tau_t) w_t,$$

and we can define the *ex ante* real interest rate R_t as follows,

$$\frac{\vartheta}{R_t} = \tilde{q}_t (1 + \pi_{t+1}).$$

Note that the *ex post* real rate will differ depending on the proportion of short and long-term bonds the household possesses in the presence of aggregate 'shocks' to the perfect foresight equilibrium path since additional capital gains/losses are possible on long-term bonds when the path of interest rates differ from what was expected.

The solution to an individual's optimization problem can be summarized by the following Proposition derived in Appendix A.

[3] For simplicity we assume that dividends are shared equally across households. It would be possible to allow dividends to vary with household labor supply or the state of the economy as in Achary and Dogra(2020). In our economy another possibility might be to allow dividends paid to individual households to vary with age, reflecting rebalancing of portfolios from equities to bonds over the life-cycle.

Proposition 1 (Household's Optimization) *In equilibrium, the optimal date t consumption and labor supply decisions of a household i born at date s are,*

$$c_t^s(i) = \mathcal{C}_t - \chi G_t + \mu_t m_t^s(i) \tag{4}$$

$$l_t^s(i) = \rho \ln(\eta_t) - \Theta_t^s - \rho\gamma\left(c_t^s(i) + \chi G_t\right) + \xi_t^s(i) \tag{5}$$

where

$$m_t^s(i) = A_t^s(i) - \varphi_t \Theta_t^s + \eta_t\left(\xi_t^s(i) - \bar{\xi}\right)$$

is 'adjusted wealth', $\mathcal{C}_t$ is a measure of common consumption, μ_t is the 'marginal propensity to consume (MPC) out of adjusted wealth and φ_t is the after-tax value of the human wealth of an individual supplying one unit of labor supply. This latter variable is used to value the income lost to retirement within households and for the population as a whole. These evolve according to

$$\frac{1}{\mu_t} = \frac{\vartheta}{R_t \mu_{t+1}} + (1 + \rho\gamma\eta_t), \tag{6}$$

$$\varphi_t = \eta_t + \frac{\vartheta}{R_t}\varphi_{t+1}, \tag{7}$$

$$\begin{aligned} \mathcal{C}_t = &-\frac{\mu_t \vartheta}{R_t \mu_{t+1}\gamma}\ln(\beta R_t) + \frac{\vartheta \mu_t}{R_t \mu_{t+1}}\mathcal{C}_{t+1} - \frac{\vartheta \mu_t}{R_t \mu_{t+1}}\frac{\gamma}{2}\mu_{t+1}^2 \eta_{t+1}^2 \sigma_{t+1}^2 \\ &- \mu_t \frac{\vartheta}{R_t}\varkappa\varphi_{t+1} + \mu_t\left(\eta_t\left(\rho\log(\eta_t) + \bar{\xi}\right) + d_t - T_t + \chi G_t\right). \end{aligned} \tag{8}$$

The household's optimization implies that their consumption equals a measure of consumption, $\mathcal{C}_t$, which only depends on aggregate variables, after adjusting for the substitutability between private and public consumption in utility, χG_t, plus a term that is idiosyncratic, $\mu_t m_t^s(i)$. This final term depends on household i's 'adjusted wealth', $m_t^s(i)$, which comprises their financial assets, $A_t^s(i)$, minus the age-dependent loss of human wealth due to retirement that period, $\varphi_t \Theta_t^s$, and the extent to which their labor income varies due to their idiosyncratic shock to labor disutility differing from the population average, $\eta_t\left(\xi_t^s(i) - \bar{\xi}\right)$. Household labor supply then depends positively on the post-tax real wage, negatively on consumption, with adjustments made for both age-dependent retirement and idiosyncratic shocks to the disutility of labor supply.

A negative shock to labor supply, $\xi_t^s(i) < \bar{\xi}$, reduces household income and results in a fall in consumption, where $\frac{\partial c_t^s(i)}{\partial \xi_t^s(i)} = \mu_t \eta_t = \mu_t(1-\tau_t) w_t$. This fall will be greater the higher the marginal propensity to consume out of adjusted wealth, μ_t, and the greater the post-tax real wage. Households are therefore more insulated from the direct impact of the shock the higher the tax rate. As a result of the fall in consumption, they will work harder, where $\frac{\partial l_t^s(i)}{\partial \xi_t^s(i)} = 1 - \gamma\rho\mu_t\eta_t = 1 - \gamma\rho\mu_t(1-\tau_t) w_t < 1$. Again, a lower marginal propensity to consume and a higher tax rate will reduce the household's desire to maintain consumption by working harder in the period of the shock. Aside from working harder, the household can also maintain consumption through borrowing. Its ability to do so is implicit in the marginal propensity to consume.

We can iterate the marginal propensity to consume out of adjusted wealth forwards to obtain,

$$\frac{\mu_t}{R_t} = \left[\sum_{s=0}^{\infty} \frac{\vartheta^s \left(1 + \rho\gamma \left(1 - \tau_{t+s}\right) w_{t+s}\right)}{\prod_{j=1}^{s+1} R_{t+j-1}}\right]^{-1}.$$

This formula is the same as Acharya et al. (2023), except it incorporates the future *post-tax* real wage rate. It indicates that the propensity to consume increases with interest rates but decreases with future post-tax wages. Therefore, after experiencing a negative idiosyncratic shock to labor supply, which reduces their adjusted wealth, $m_t^s(i)$, households can maintain consumption closer to $\mathcal{C}_t$ when the marginal propensity to consume is low. This occurs when interest rates are low, making borrowing to smooth consumption less costly, or when post-tax wages are expected to be higher in the future, making it less expensive to repay any borrowing. Additionally, the presence of the tax rate implies that a lower tax rate makes it less costly (in utility terms) to increase future labor supply to pay off any debt incurred to smooth consumption. Thus, future distortionary taxation inhibits self-insurance, although high tax rates at the time of the shock mitigate its direct impact, as part of the lost income would have been taxed anyway.

Meanwhile, the component of household consumption driven by aggregate variables, $\mathcal{C}_t$, can be iterated forwards to obtain,

$$\begin{aligned}\mathcal{C}_t = &-\frac{1}{\gamma}\sum_{s=0}^{\infty} Q_{t+s,t}\frac{\mu_t}{\mu_{t+s}}\ln(\beta R_{t+s}) - \frac{\gamma\mu_t}{2}\sum_{s=0}^{\infty} Q_{t+s,t}\mu_{t+s}^2 w_{t+s}^2 (1-\tau_{t+s})^2\sigma_{t+s}^2 \\ &+ \mu_t \sum_{s=0}^{\infty} Q_{t+s,t}\overline{y}_{t+s} - \varkappa\mu_t \sum_{s=1}^{\infty} Q_{t+s,t}\varphi_{t+s}.\end{aligned}$$

The first term has the same interpretation as in Acharya and Dogra (2020), capturing the impact of variations in interest rates relative to the impatience of households. If interest rates are typically higher than the rate of time preference, current consumption will be lower as households increase savings and cut current consumption. The discount factor, $Q_{t+s,t} = \frac{\vartheta^s}{\prod_{j=0}^{s-1} R_{t+j}}$, accounts for both the interest rate on financial assets and the probability of death, $1-\vartheta$. The second term is attributable to precautionary savings. A higher variance of idiosyncratic shocks, σ_{t+s}^2, increases the variance of post-tax income, $w_{t+s}^2(1-\tau_{t+s})^2\sigma_{t+s}^2$, which, after applying the marginal propensity to consume, captures the variance in consumption across households, $\mu_{t+s}^2 w_{t+s}^2(1-\tau_{t+s})^2\sigma_{t+s}^2$. The third term represents the discounted value of per capita post-tax income from labor, dividends, and transfers, after adjusting for the utility generated by public consumption, $\overline{y}_t = \left(\eta_t \left(\rho \log(\eta_t) + \bar{\xi}\right) + d_t - T_t + \chi G_t\right)$. Lastly, the equation includes the discounted value of the income lost due to the gradual retirement of the population throughout their working lives. Taxation affects this measure of aggregate consumption through its impact on the marginal propensity to consume, as discussed above, positively by reducing the variance of post-tax income but negatively by reducing the level of post-tax income and, therefore, the discounted value of that income.

It follows that net income (3) can be written as

$$y_t^s(i) = \eta_t \left(\rho \log(\eta_t) + \xi_t^s(i)\right) - \eta_t \Theta_t^s - \rho\gamma\chi\eta_t G_t - \rho\gamma\eta_t c_t^s(i) + d_t - T_t. \tag{9}$$

Aggregation is detailed in Appendix B, where aggregation of the household budget constraint yields,

$$\frac{\vartheta}{R_t} A_{t+1} = \vartheta A_t + y_t - c_t, \tag{10}$$

with,

$$A_t = \frac{\left(1 + \varrho \tilde{P}_t^M\right) a_t^L + a_t^S}{1 + \pi_t},$$

and a_t^J is an aggregation of long term $(J = L)$ and short term $(J = S)$ bonds.

Straightforward aggregation of income (9) yields,

$$y_t = \eta_t \left(\rho \log(\eta_t) + \bar{\xi}\right) - \frac{\varkappa\vartheta}{(1-\vartheta)} \eta_t - \rho\gamma\eta_t \chi G_t - \rho\gamma\eta_t c_t + d_t - T_t,$$

and aggregation of (4),

$$c_t = \mathcal{C}_t - \chi G_t + \mu_t \vartheta \left(A_t - \frac{\varkappa}{1-\vartheta} \varphi_t\right).$$

This latter expression indicates that per capita consumption equals the consumption measure, $\mathcal{C}_t$, driving individual household consumption in (4), after adjusting for the substitutability between private and public consumption, χG_t, and the extent to which, in aggregate, households have successfully saved for retirement. $A_t > \frac{\varkappa}{1-\vartheta}\varphi_t$ implies that household financial wealth exceeds the loss of human wealth due to retirement across the population.

Aggregated first order conditions for the individuals' problem yield the following relationships, derived in Appendix C.

Proposition 2 (Aggregated Households' Optimization) *In equilibrium, the optimal date t* *aggregate total consumption and labor supply decisions are,*

$$x_t = -\frac{1}{\gamma}\log(\beta R_t) + x_{t+1} + \mu_{t+1}(1-\vartheta)A_{t+1} - \frac{\gamma}{2}\mu_{t+1}^2\eta_{t+1}^2\sigma_{t+1}^2 - \varkappa\mu_{t+1}\varphi_{t+1}, \tag{11}$$

$$n_t = \rho\log\eta_t - \frac{\varkappa\vartheta}{1-\vartheta} + \bar{\xi} - \rho\gamma x_t, \tag{12}$$

$$x_t = c_t + \chi G_t, \tag{13}$$

$$\mathcal{C}_t = x_t - \mu_t\vartheta\left(A_t - \frac{\varkappa}{(1-\vartheta)}\varphi_t\right). \tag{14}$$

The dynamics of x_t resemble that of consumption in a representative agent model, but with notable differences. Typically, consumption is expected to grow whenever the interest rate exceed the rate of time preference, $\beta R_t > 1$. In other words, consumption jumps down when interest rates unexpectedly rise, as the discounted value of future post-tax income across the economy falls. Consumption then recovers as interest rates return to normal levels. However, there is an additional term, $\mu_{t+1}(1-\vartheta)A_{t+1}$, attributable to the aggregation across finitely-lived generations. This term would not exist if households were infinitely lived and $\vartheta = 1$. Instead, finite lives imply that government debt (which is mapped to households assets as $B_t = \vartheta A_t$) are net assets for households. Households currently alive do not expect to pay for all the surpluses backing government debt, implying that any increase in those assets increases consumption. As above, the term $\frac{\gamma}{2}\mu_{t+1}^2\eta_{t+1}^2\sigma_{t+1}^2$ measures the variance of consumption across households due to idiosyncratic shocks, providing a motive for precautionary saving, which in turn reduces current consumption. Finally, consumption is reduced by the ongoing loss of post-tax income due to retirement.

It is helpful to consider the steady-state of this relationship to see how these additional factors influence interest rates,

$$\frac{1}{\gamma}\log(\beta R) = \mu(1-\vartheta)(A - \frac{\varkappa}{1-\vartheta}\varphi) - \frac{\gamma}{2}\mu^2\eta^2\sigma^2.$$

In the absence of idiosyncratic risk or finite lives, the steady-state interest rate in a representative agent economy would be consistent with household preferences, $\beta R = 1$. However, the desire for precautionary savings drive down the steady-state interest rate relative to these preferences, while the accumulation of assets beyond what is needed to fund retirement in an OLG economy, $A > \frac{\varkappa}{1-\vartheta}\varphi$, raises interest rates. If the government could provide sufficient assets for households to satiate their desire for precautionary savings and their need to smooth consumption in retirement, then the steady-state interest rate would equal the households rate of time preference, provided,

$$B - \frac{\varkappa}{1-\vartheta}\varphi = \frac{1}{2}\frac{\vartheta}{1-\vartheta}\gamma\mu\eta^2\sigma^2. \tag{15}$$

This 'golden rule' benchmark becomes relevant when considering Ramsey policy below.

2.2 Firms

There is a continuum of monopolistically competitive firms. Each firm produces a differentiated product according to the production technology,

$$Y_t(j) = z_t n_t(j), \tag{16}$$

where z_t is the level of aggregate productivity.

They face a cost of price adjustment *a la* Rotemberg (1982). In the absence of aggregate risk, firm j solves the following optimization problem

$$\max_{P_t(j)} \sum_{t=0}^{\infty} \beta^t \left(\left(\frac{P_t(j)}{P_t} Y_t(j) - w_t n_t(j) \right) - \frac{\Phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t \right),$$

subject to monopolistic demand for its product,

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon_t} Y_t,$$

and production function (16).

The profit optimization yields (see Appendix D) the following nonlinear Phillips curve,

$$\pi_t(1+\pi_t) = \frac{1-\varepsilon_t+\varepsilon_t \frac{w_t}{z_t}}{\Phi} + \beta \pi_{t+1}(1+\pi_{t+1}) \frac{Y_{t+1}}{Y_t},,$$

and any profit is distributed as a dividend,

$$d_t = (Y_t - w_t n_t) - \frac{\Phi}{2} \pi_t^2 Y_t. \tag{17}$$

2.3 Government

The government issues nominal long term and short term bonds, for which the maturity matches that of the actuarial bonds used by households. The government budget constraint in nominal terms is

$$P_t^M \mathcal{B}_{t+1}^L + q_t \mathcal{B}_{t+1}^S = \left(1 + \varrho P_t^M\right) \mathcal{B}_t^L + \mathcal{B}_t^S + P_t G_t - \tau_t P_t w_t n_t - P_t T_t$$

where P_t^M is price of long-term bonds, and q_t is price of short term bonds. As noted above, the lump sum taxes, $P_t T_t$, are generally set to zero and only used as a replacement for distortionary tax revenues, $\tau_t P_t w_t n_t$, when we wish to remove the impact of distortionary taxation on optimal policy.

This can be re-written in real terms,

$$(1+\pi_{t+1}) q_t B_{t+1} = B_t + G_t - \tau_t w_t n_t - T_t \tag{18}$$

where

$$B_t = \frac{\left(\left(1+\varrho P_t^M\right) b_t^L + b_t^S\right)}{(1+\pi_t)}$$

and

$$b_t^J = \frac{\mathcal{B}_t^J}{P_{t-1}}, J \in \{L, S\}.$$

2.4 Financial Intermediaries

Financial intermediaries trade actuarial and government bonds. The real profit of intermediaries is the difference between total bonds and total amount of actuarial bonds in the economy in $t+1$,

$$\Pi = \left(1+\varrho P_{t+1}^M\right) b_{t+1}^L + b_{t+1}^S - \left(1+\varrho \tilde{P}_{t+1}^M\right) \vartheta a_{t+1}^L - \vartheta a_{t+1}^S, \tag{19}$$

where b_{t+1}^J are total government bonds and ϑa_{t+1}^J are total actuarial bonds at time $t+1$, i.e. $\vartheta a_{t+1}^J = (1-\vartheta)\sum_{s=-\infty}^{t+1} \vartheta^{t+1-s} \int_0^1 a_{t+1}^{J,s}(i)\, di$.

The intermediaries maximize (19) subject to the constraint,

$$-\tilde{P}_t^M a_{t+1}^L - \tilde{q}_t a_{t+1}^S + P_t^M b_{t+1}^L + q_t b_{t+1}^S \leqslant 0. \tag{20}$$

and the optimization yields

$$\frac{1}{\tilde{q}_t} = \frac{\left(1+\varrho \tilde{P}_{t+1}^M\right)}{\tilde{P}_t^M}, \tag{21}$$

$$\tilde{q}_t = \vartheta q_t, \tag{22}$$

$$\frac{1}{q_t} = \frac{\left(1+\varrho P_{t+1}^M\right)}{P_t^M}, \tag{23}$$

such that the intermediaries' profits are zero and the *ex ante* returns on short and long-bonds are equalized. It is important to note, however, that this does not imply that the *ex post* real interest rates will be equalized in the presence of one-off shocks to the perfect foresight equilibrium path.

We denote the short-term nominal interest rate as,

$$\frac{1}{1+i_t} = q_t,$$

and the real interest rate is,

$$R_t = \frac{\vartheta}{\tilde{q}_t\left(1+\pi_{t+1}\right)} = \frac{1}{q_t\left(1+\pi_{t+1}\right)} = \frac{1+i_t}{1+\pi_{t+1}}.$$

2.5 Market Clearing

We use households' budget constraints (10), the government's budget constraint (18), profits of financial intermediaries (19), aggregate income (3) and the profits of monopolistic firms (17) to obtain the resource constraint,

$$Y_t = c_t + G_t + \frac{\Phi}{2}\pi_t^2 Y_t. \tag{24}$$

Finally, using (19) we can rewrite consumption decision (14) in terms of aggregate debt,

$$\mathcal{C}_t = c_t + \chi G_t - \mu_t \left(B_t - \frac{\vartheta \varkappa}{1-\vartheta}\varphi_t \right). \tag{25}$$

2.6 Private Sector Equilibrium

The dynamic system which determines private sector equilibrium $\{x_t, Y_t, \pi_t, \eta_t, w_t, B_t, P_t^M, R_t, \mu_t, \varphi_t, \sigma_t^2\}$ given policy $\{i_t, G_t, T_t, \tau_t\}$ and deterministic disturbances z_t and ε_t can be written as follows,

$$x_t = -\frac{1}{\gamma}\log(\beta R_t) + x_{t+1} + \frac{(1-\vartheta)}{\vartheta}\mu_{t+1}B_{t+1} - \frac{\gamma}{2}\mu_{t+1}^2\eta_{t+1}^2\sigma_{t+1}^2 - \varkappa\mu_{t+1}\varphi_{t+1}, \tag{26}$$

$$\pi_t(1+\pi_t) = \frac{1-\varepsilon_t+\varepsilon_t\frac{w_t}{z_t}}{\Phi} + \beta\pi_{t+1}(1+\pi_{t+1})\frac{Y_{t+1}}{Y_t}, \tag{27}$$

$$\frac{1}{\mu_t} = \frac{\vartheta}{R_t\mu_{t+1}} + (1+\rho\gamma\eta_t), \tag{28}$$

$$(1+\pi_{t+1})\, q_t B_{t+1} = B_t + G_t - \tau_t w_t n_t - T_t, \tag{29}$$

$$\frac{Y_t}{z_t} = \rho\log\eta_t + \bar{\xi} - \varkappa\frac{\vartheta}{1-\vartheta} - \rho\gamma x_t, \tag{30}$$

$$\eta_t = (1-\tau_t)\, w_t, \tag{31}$$

$$Y_t = x_t + (1-\chi)\, G_t + \frac{\Phi}{2}\pi_t^2 Y_t, \tag{32}$$

$$P_t^M R_t = \frac{\left(1+\varrho P_{t+1}^M\right)}{(1+\pi_{t+1})}, \tag{33}$$

$$R_t = \frac{1+i_t}{1+\pi_{t+1}}, \tag{34}$$

$$\varphi_t = \eta_t + \frac{\vartheta}{R_t}\varphi_{t+1}, \tag{35}$$

$$\sigma_t^2 = \sigma^2\exp\left(2\phi\left(Y_t - Y\right)\right), \tag{36}$$

where in the last equation, following Acharya and Dogra (2020), we assume that risk is procyclical if $\phi > 0$ and countercyclical if $\phi < 0$.

3 Social Welfare Function and Optimal Policy

3.1 Social Welfare Objective

We define the social welfare function at time $t = 0$ as,

$$\mathbb{W}_0 = (1-\vartheta)\left(\sum_{s=-\infty}^{0} \vartheta^{-s} \int_0^1 W_0^s(i)\, di + \sum_{s=1}^{\infty} \beta^s \int_0^1 W_s^s(i)\, di\right), \tag{37}$$

where the first term represents the utility of generations that are alive at time zero. The currently living generations are treated equally after accounting for their relative size. The second term represents the utility of unborn generations ($s > 0$), and the utility of each such generation is discounted with weight β^s. Appendix F shows that this welfare measure can be written as follows,

$$\mathbb{W}_0 = \sum_{t=0}^{\infty} \beta^t \mathbb{U}_t,$$

where

$$\mathbb{U}_t = -\frac{1}{\gamma}\left(1 + \gamma\rho\eta_t\right) e^{-\gamma x_t} S_t,$$

and S_t satisfies the recursion,

$$S_t = \left(\vartheta e^{-\frac{\gamma}{\vartheta} W_t} S_{t-1} + 1 - \vartheta\right) e^{\gamma W_t} e^{\frac{1}{2}\gamma^2 \mu_t^2 \eta_t^2 \sigma_t^2}. \tag{38}$$

Here,

$$W_t = \mu_t \left(B_t - \frac{\vartheta \varkappa}{(1-\vartheta)} \varphi_t\right) \tag{39}$$

measures the extent to which society has succeeded in financing its retirement. It extends the form of the welfare function considered in Acharya et al. (2023) by accounting for intergenerational inequality as well as the distribution of consumption driven by idiosyncratic shocks. The first part of the social welfare function captures the utility generated by per capita levels of private and public consumption, less the disutility of labor supply. The second element adjusts that measure for the welfare effects of inequality, driven by both idiosyncratic shocks and the distribution of consumption and labor supply across generations due to the endogenous accumulation of assets and age-related withdrawal from the labor market.

To gain intuition for these effects, it is helpful to consider the steady state of the measure of the social costs of inequality, S_t. In the steady state, the expression becomes,

$$S = \frac{(1-\vartheta)\, e^{\gamma W} e^{\frac{1}{2}\gamma^2 \mu^2 \eta^2 \sigma^2}}{1 - \vartheta e^{-\gamma \frac{(1-\vartheta)}{\vartheta} W} e^{\frac{1}{2}\gamma^2 \mu^2 \eta^2 \sigma^2}}.$$

Taking the partial derivative of this measure of inequality with respect to W yields,

$$\frac{\partial S}{\partial W} = \gamma S \left(1 - S e^{-\gamma \frac{W}{\vartheta}}\right).$$

The choice of W that would minimize steady-state inequality, treating $\mu^2\eta^2$ as given, would be,

$$W = \frac{1}{2}\left(\frac{\vartheta}{1-\vartheta}\right)\gamma\mu^2\eta^2\sigma^2.$$

This would not eliminate inequality but would facilitate a degree of self-insurance by providing assets for households to undertake both precautionary savings and retirement savings. Recall that interest rates are consistent with the household's rate of time preference when this exact condition holds – see equation (15). Therefore, this level of debt is also the one that ensures the steady-state equilibrium real interest rate equals the households' rate of time preference, $R = \beta^{-1}$. However, it is important to note that this will not be consistent with the Ramsey optimum since the Ramsey policymaker will also consider the endogeneity of $\mu^2\eta^2$ and be concerned with efficiency as well as equity. Increasing W increases the marginal propensity to consume, which reduces the ability of households to borrow against future income in the face of negative idiosyncratic shocks. Therefore, in steady-state, even a policymaker concerned only with inequality might not issue this level of debt as it reduces households' ability to respond to idiosyncratic shocks. We will see below that the Ramsey policymaker delivers a level of debt that falls short of ensuring $R = \beta^{-1}$, even if their objective were solely to minimize inequality. Raising W also results in higher taxes to sustain the higher debt level. Since taxation is distortionary, the policy maker would then wish to reduce debt further if they also had a concern for efficiency. We explore these trade-offs in the next section.

We can also consider special cases to gain further insight. If there were no idiosyncratic shocks, but there was potential intergenerational inequality due to age-related retirement, then this would imply steady-state inequality of:

$$S = \frac{(1-\vartheta)\, e^{\gamma W}}{\left(1 - \vartheta e^{-\gamma \frac{(1-\vartheta)}{\vartheta} W}\right)}, \tag{40}$$

and we could eliminate inequality if $W = 0$, so that $B = \frac{\vartheta\varkappa}{(1-\vartheta)}\varphi$. In other words, by issuing sufficient debt to absorb the desire to save for retirement and ensuring interest rates are the same as the households' rate of time preference, the policymaker can eliminate steady-state intergenerational inequality. Households would save by buying government bonds to ensure they had sufficient assets to maintain consumption even as their income falls due to retirement. This would achieve consumption equality across generations.

If, however, we reintroduce idiosyncratic shocks while the policy maker balanced the steady-state level of debt with the discounted value of the income lost through phased retirement, such that $W = 0$, then the inequality measure would reduce to:

$$S = \frac{(1-\vartheta)\, e^{\frac{1}{2}\gamma^2\mu^2\eta^2\sigma^2}}{1-\vartheta e^{\frac{1}{2}\gamma^2\mu^2\eta^2\sigma^2}}, \tag{41}$$

so that when $\sigma^2 > 0$, then $S > 1$ due to the costs of idiosyncratic shocks considered by Acharya and Dogra (2020). This situation would imply that $R < \beta^{-1}$ as households still have an additional motive to undertake precautionary savings, beyond saving for retirement. In the absence of sufficient assets to fulfill that desire, interest rates will lie below the households' rate of time preference. We shall explore where the Ramsey policy maker chooses to set the steady-state level of debt in light of these trade-offs in the next section.

3.2 Optimal Policy

The policymaker seeks to maximize

$$\sum_{t=0}^{\infty} \beta^t \left(-\frac{1}{\gamma}\left(1+\gamma\rho\eta_t\right) S_t^{\xi} \exp\left(-\gamma x_t\right)\right), \tag{42}$$

subject to the system describing the private sector equilibrium (26)-(36), the recursion of inequality measure (38), and the definition (39).

In the policy objective (42), we can set the parameter ξ to either zero or one. When it is one, the policymaker cares about both equity and efficiency, consistent with the micro-founded social welfare function derived above. When ξ is zero, the policymaker is concerned only with efficiency, not equity. We shall also consider another scenario in which the policy maker cares only about equity and aims to minimize

$$\sum_{t=0}^{\infty} \beta^t S_t. \tag{43}$$

We assume that the policymaker has access to a commitment technology, and all first order conditions are presented in Appendix G.

4 Calibration

The model is calibrated to a quarterly frequency for the US economy. Most parameter calibrations are standard and generally follow those in Acharya et al. (2023). We calibrate the household discount rate to $\beta = (1.02)^{-1/4}$, aligning it with a real interest rate of 2% per annum, which is the average in the US over the Great Moderation period (1984-2021). The coefficient of relative risk aversion is set to $\gamma = 2$, based on evidence in Hall (1988), Campbell and Mankiw (1989) and Attanasio and Weber (1993, 1995). The Frisch elasticity of substitution is set at $\rho = 1/2$ following empirical evidence in Fagereng et al. (2017) and Christelis et al. (2015).

Fiscal parameters are based on data from the same period. Specifically, the parameter ϱ is set to match the maturity of government debt to 20 quarters, closely aligning with the 5.4 years observed in the data (IMF, 2016). The parameter G is set to generate a spending share $G/Y = 0.15$, as reported in IMF IFS data.[4] The relative weight on government consumption in utility, χ, is set to 0.05, which is a free parameter chosen to ensure that government expenditure is not fully wasted. The elasticity of substitution between goods, ϵ, is set to 11 based on evidence in Chari et al. (2000), corresponding to a markup of 10%.

Our model incorporates nominal rigidities following Rotemberg (1982). Most recent papers in the macroeconomics literature, which calibrate their frameworks for the US economy, assume that prices change every 10 months (see Klenow and Kryvtsov, 2008, Nakamura and Steinsson, 2008 and Klenow and Malin, 2010). As the Rotemberg (1982) and Calvo (1983) models generate isomorphic linearized New Keynesian Phillips curves, the equivalent Rotemberg model parameter is $\Phi = 106.4$. The parameter $\bar{\xi}$ is set to 2, which normalizes output to one in the special case of a monetary model with a labor market subsidy that offsets monopolistic distortions. Therefore, an equilibrium level of output which differs from one measures the extent to which the economy differs from its efficient level.

We choose the survival rate to be consistent with an average lifespan of 80 years, as reported by SSA data.[5] The declining labor supply efficiency parameter, $\varkappa$, is chosen to be consistent with 20 years of retirement, in line with the US data over the last 50 years.[6] We follow Guvenen et al. (2014), who document the standard deviation of the one-year growth rate of log earnings is about 0.5. This yields $\sigma = 0.33$ for the baseline calibration.

Finally, we calibrate the persistence of deterministic processes for productivity and elasticity of substitution to be 0.95 and 0.9, respectively. This again follows Acharya et al. (2023), who adopt the empirical estimates of Bayer, Born, and Luetticke (2024).

[4] The relevant data series are NGDP_XDC and NCGG_XDC.

[5] See Period Life Table at www.ssa.gov.

[6] See https://crr.bc.edu/wp-content/uploads/2024/04/Average-retirement-age_2021-CPS.pdf

5 Ramsey Steady State

In general, we need to solve the steady state of the Ramsey policymaker's problem numerically. However, there are some interesting special cases which can be solved analytically. The first is the case where there are no idiosyncratic shocks ($\sigma^2 = 0$) and the fiscal policy instrument is a lump-sum rather than distortionary tax. In this scenario, the steady state of the economy under the Ramsey plan is described by the following proposition.

Proposition 3 *With access to lump-sum taxes as a policy instrument and in the absence of idiosyncratic shocks, the Ramsey steady state is given by:*

$$R = \frac{1}{\beta},\ W = 0,\ S = 1,\ \pi = 0,\ w = \eta = \frac{\varepsilon - 1}{\varepsilon},\ \varphi = \frac{R\eta}{(R-\vartheta)},\ B = \frac{\vartheta\varkappa}{(1-\vartheta)}\varphi,\ \textit{and}\ P^M = \frac{1}{R-\varrho}$$

Here, the policymaker would choose to eliminate inequality by issuing sufficient debt to facilitate households' saving for retirement, ensuring that consumption is constant in steady-state: $B = \frac{\vartheta\varkappa}{(1-\vartheta)}\varphi$. As discussed above, this ensures that the steady-state real interest rate is consistent with the households' rate of time preference, $R = \beta^{-1}$. Issuing more (or less) debt than this would drive interest rates above (or below) the households' rate of time preference, resulting in consumption rising (or falling) over an individual household's life, thereby creating undesirable intergenerational inequality.

If we then maintain the assumption that there are no idiosyncratic shocks ($\sigma^2 = 0$), but the available fiscal instrument is a distortionary tax on labor income, then the Ramsey policymaker would only wish to eliminate intergenerational inequality if there is no age-related increase in the disutility of supplying labor ($\varkappa = 0$).

Proposition 4 *When the available fiscal policy instrument is a distortionary tax on labor income, then $R = \frac{1}{\beta}$ only holds in the Ramsey steady state in the absence of both idiosyncratic shocks ($\sigma^2 = 0$) and retirement ($\varkappa = 0$).*

This special case implies that the Ramsey policymaker does not issue debt in the steady state. With retirement, in order to ensure that interest rates are consistent with the households' rate of time preference, the policymaker would need to issue debt, which becomes costly when debt service costs must be financed through distortionary taxation.

6 Discussion

6.1 Policy Trade-offs

We begin our numerical analysis by exploring the steady state of the model and the trade-offs between equity and efficiency faced by the Ramsey policymaker. In the first column of Table 1, we present the Ramsey steady state of our benchmark economy, which features jointly optimal monetary and fiscal policy. The fiscal policy instrument is a distortionary labor income tax. Households are subject to idiosyncratic income shocks and must plan for a gradual withdrawal from the labor market over their lifetimes. The second column repeats this exercise but removes fiscal policy ($G = 0$, $B = 0$, and $\tau = 0$), leaving the Ramsey planner with only monetary policy as a tool to affect the equilibrium. The third column returns to the benchmark economy but investigates the steady state that would occur if the Ramsey policy did not prioritize addressing inequality. The final column considers the opposite extreme, where the policymaker focuses solely on equity, seeking to minimize $\sum_{t=0}^{\infty} \beta^t S_t$.

These results are mirrored in Figure 1, which plots the steady-state value of variables given an equilibrium without inflation ($\pi = 0$) but conditional on a given steady-state debt-to-GDP ratio. Markers are placed on this line, representing equilibrium outcomes under the Ramsey policy (star), Ramsey policy concerned only with per capita averages (hollow circle), Ramsey policy focused solely on inequality (inverted triangle), and the steady state of the model without any fiscal policy (cross). These markers correspond to columns 1, 3, 4 and 2 of Table 1, respectively. The figure explores how equilibrium outcomes change as debt policy changes and offers a visual representation of the extent to which Ramsey policy resolves the trade-off between equity and efficiency.

The first point to note, when comparing columns (1) and (2) or stars with crosses, is the ability of fiscal policy to mitigate inequality. Without fiscal policy, individual households' efforts to save for both precautionary reasons and retirement drive down the equilibrium real interest rate below the households' rate of time preference ($R < \beta^{-1}$). Since there are no assets available in aggregate for households to hold, the desire to save for these two motives forces equilibrium returns on saving to fall, discouraging saving behavior. As a result, households do not accumulate enough wealth to maintain consumption in retirement, leading to a gradual decline in consumption as they withdraw from the labor market. Thus, the high level of inequality observed in the monetary-policy-only economy is largely the result of significant intergenerational inequality. With the introduction of fiscal policy, the government issues a substantial amount of debt, facilitating household saving

		Ramsey policy	No fiscal policy	Ramsey policy: Efficiency, $\xi = 0$	Ramsey policy: Equity
		(1)	(2)	(3)	(4)
Net real interest rate, % per annum	R	1.995%	1.956%	1.980%	1.998%
Propensity to consume	μ	0.005038	0.00456	0.005004	0.005045
Per Capita Consumption	c	0.836	0.962	0.837	0.835
Output	Y	0.984	0.962	0.985	0.983
Inflation rate, % pa	π	0%	0%	0%	0%
Tax rate	τ	0.177	–	0.172	0.178
Debt minus Lost Retiral Income	W	-0.0012	-0.0134	-0.0059	-0.0004
Debt to output ratio	$\frac{P^M B}{4Y}$	54%	–	31%	58%
Inequality	S	1.00080	1.00106	1.00087	1.00079

Table 1: Steady State Values in PRANK Economy.

and causing interest rates to rise. This allows households to save more effectively for retirement, reducing intergenerational inequality. However, the amount of debt issued is insufficient to fully satisfy households' desire to save for retirement, even before considering their additional need for precautionary savings in the face of idiosyncratic shocks. As a result, interest rates do not reach the households' rate of time preference but fall slightly short. This shortfall is partly due to the efficiency-reducing distortions created by the taxes needed to service the debt, implying a trade-off between equity and efficiency for the Ramsey planner.

The relative position of the markers implies that the trade-off between efficiency and equity is resolved firmly in favor of equity, as confirmed by comparing the final two columns of Table 1. Given the decline in labor income as households age and the subsequent desire to save in anticipation of this 'retirement', the policymaker concerned with inequality wishes to issue debt to facilitate saving for retirement and prevent the significant intergenerational inequality that would emerge if there were insufficient assets to smooth consumption over the life cycle. This leads to steady-state debt levels of 53% of GDP under Ramsey policy, which falls only slightly short of the 57% debt ratio that would occur if the policymaker were solely concerned with inequality. It is interesting to note that even the debt level associated with a desire to minimize inequality is less than what is needed to drive interest rates to 2%. Since $W < 0$, they fail to supply enough assets to support households' desire to save for both retirement and precautionary reasons. The policymaker issues slightly less debt than this benchmark level even when focused

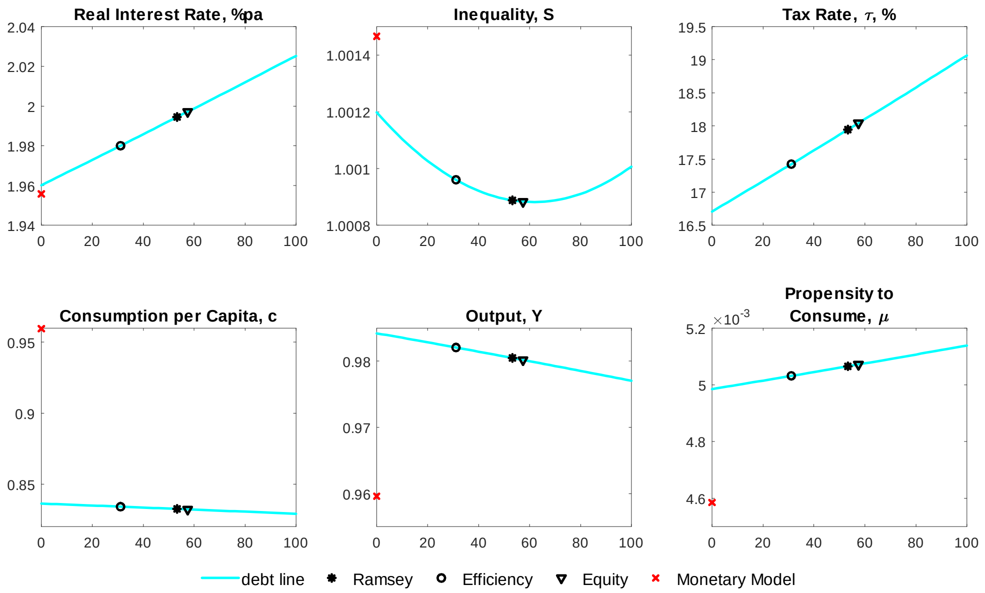

Figure 1: Debt-to-GDP Ratio and Steady-State Outcomes. The solid line is the steady state level in the HANK model where monetary and fiscal policy jointly set zero inflation and a desired level of government debt, the latter is shown on a horizontal axis. Special cases of optimal monetary and fiscal policy – depicted with different markers – will lie on this line, as – for an optimal level of government debt – all these policies also deliver zero inflation. The zero-inflation outcome in the monetary model is given as a benchmark.

solely on inequality, as lower interest rates help households smooth consumption in the face of idiosyncratic shocks. In other words, while the main driver of inequality is earnings over the life cycle, the additional inequality caused by idiosyncratic shocks leads a policymaker focused solely on mitigating inequality to pull back slightly from the golden rule level of debt.

In contrast to the case where minimizing inequality is the primary policy objective, a policymaker concerned with efficiency alone (column 4 and hollow circles) would wish to limit debt issuance to 31% of GDP—well below the 53% level adopted by the Ramsey policymaker maximizing social welfare—in order to lower the output losses due to distortionary taxation. The policymaker does not go further by lowering debt beyond this, as the fiscal consolidation implied by further debt reduction would be more costly than the steady-state gain from reduced debt-service costs.

Figure 1 also allows us to explore policies beyond those described in Table 1. As we increase debt levels from zero, this steadily raises the steady-state real interest rate and the taxes needed to service the debt. This, in turn, leads to the crowding out of private sector consumption as tax distortions reduce output. However, at the same time, higher debt levels reduce inequality. As discussed above, this is largely due to the fact that issuing debt provides households with an asset to hold to finance consumption in retirement, thereby mitigating intergenerational inequality. Inequality falls until debt levels reach 53% of GDP, at which point, although households still suffer a loss of consumption in retirement (since $R < \beta^{-1}$ at this point), the higher interest rate inhibits households' ability to smooth consumption in the face of idiosyncratic shocks, thereby increasing inequality by as much as facilitating retirement savings reduces it. Beyond that point and until $R = \beta^{-1}$, more debt increases inequality as households are less able to respond to idiosyncratic shocks, even though intergenerational inequality continues to decline. When debt rises enough to imply that $R > \beta^{-1}$, the returns to savings are now so high that households save more than they need for retirement, leading to an increase in individual household consumption as they age, even although they also withdraw from the labor market over time.

As discussed in Propositions 3-4, the desirability of achieving the golden rule interest rate of $R = \beta^{-1}$ depends on the existence of idiosyncratic shocks, the availability of lump-sum taxation, and the need to save for retirement. Table 2 explores these factors further. The first two columns consider the case where the policymaker has access to lump-sum taxation, with and without labor force participation declining with age. The final two columns do the same, but in these, the fiscal instrument is a distortionary tax rate. The figures in brackets are for the same economy, but without idiosyncratic shocks ($\sigma^2 = 0$). We can see that without idiosyncratic risk, the policymaker issues sufficient debt to ensure $R = \beta^{-1}$ and eliminate inequality across the first three columns, but does not do so when saving for retirement becomes relevant. The reason is that retirement requires a sizeable issuance of debt to avoid household saving driving down the equilibrium interest rate, but that debt must be serviced through increases in distortionary taxation, which are costly in terms of efficiency.

When we consider the same variants, but with idiosyncratic risk, the policymaker always fails to drive interest rates to $R = \beta^{-1}$. In fact, they never issue sufficient debt to allow households to maintain consumption in retirement, even when taxes are lump-sum. The reason is that there is a need to suppress interest rates below the households' rate of time preference. The lower interest rate then facilitates individual households' ability to borrow to maintain consumption in response to negative idiosyncratic shocks. When taxes are distortionary, the costs of issuing debt are higher still, further inhibiting the debt issuance of the policymaker.

This analysis also lets us to quantify how inequality diminishes social welfare. Since social

		Lump Sum Tax		Income Tax	
		$\varkappa = 0$	$\varkappa > 0$	$\varkappa = 0$	$\varkappa > 0$
		(1)	(2)	(3)	(4)
Net real interest rate, % p.a.	R	1.997% (2%)	1.997% (2%)	1.998% (2%)	1.995% (1.997%)
Propensity to consume	μ	0.00462 (0.00462)	0.00462 (0.00462)	0.00500 (0.00501)	0.00503 (0.00504)
Consumption per capita	c	0.8985 (0.8985)	0.8844 (0.8844)	0.8543 (0.8542)	0.8356 (0.8355)
Output	Y	1.0465 (1.0465)	1.0324 (1.0324)	1.0023 (1.0022)	0.9836 (0.9835)
Inflation rate, % p.a.	π	0% (0%)	0% (0%)	0% (0%)	0% (0%)
Tax rate	τ	– (–)	– (–)	0.162 (0.162)	0.177 (0.178)
Lump Sum Taxes	T	0.148 (0.148)	0.162 (0.162)	– (–)	– (–)
Debt minus Lost Retiral Income	W	−0.0005 (0.0)	−0.0005 (0.0)	−0.0003 (0.0)	−0.0012 (−0.0010)
Debt to output ratio,	$\frac{P^M B}{4Y}$	−2.5% (0%)	67.3% (69.8%)	−1.6% (0%)	54.2% (55.7%)
Inequality	S	1.00098 (1.0000)	1.00098 (1.0000)	1.00081 (1.0000)	1.00079 (1.000002)

Table 2: Steady State Values in PRANK Economy. Corresponding RANK values are in paretheses.

welfare is reduced by the factor S_t the extent to which $S > 1$ measures how much social welfare is reduced as a result of inequality conditional on the aggregate per capita values of all other variables. Thus inequality reduces social welfare by 0.08% under the Ramsey policy which is only slightly more than when the policy maker cares solely for inequality, 0.079%, but significantly less than when the policy maker cares only for efficiency, 0.087%. This reflects the relative positions of policies under different policy maker objectives in Figure 1. For any variable we could consider, Ramsey policy is closer to a policy which cared only for inequality than one which cared only for efficiency.

6.2 Dynamic Responses

We now turn to consider the policy response to shocks. Since there is no aggregate risk in our economy, we are dealing with the perfect foresight equilibrium path in response to a one-off auto-correlated positive increase in productivity – a positive aggregate productivity shock.[7] It is well known that in the standard New Keynesian monetary model without heterogeneity, after ensuring

[7] We also considered reduction in elasticity of substitution between differentiated goods. These results are available upon request.

an efficient steady state, such shocks are subject to the 'divine coincidence' (see Blanchard and Galí, 2007), implying that adjusting interest rates to maintain output at its natural level can be achieved without generating any inflation. This result does not hold in our heterogeneous agent

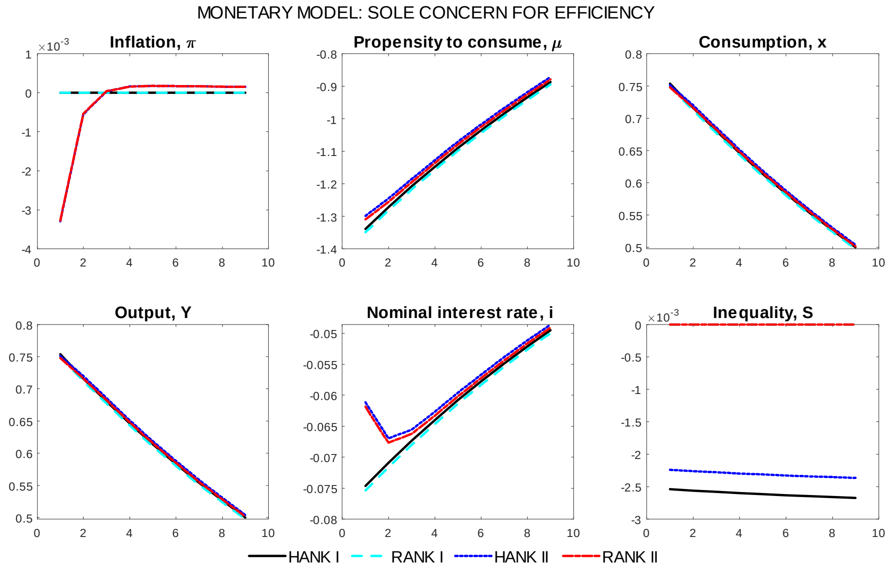

Figure 2: Dynamic responses to a positive technology shock. Hank I is our heterogeneous agent economy without fiscal policy, but with a production subsidy to offset monopolistic competition distortion. Rank I removes phased retirement, $\varkappa = 0$, and idiosycratic shocks, $\sigma^2 = 0$. Hank II and Rank II remove the production subsidy. In all scenarios there is no policy concern for equity.

economy. In fact, to return to this familiar result, we need to (i) remove the fiscal elements of our model (debt, distortionary taxation, and government consumption), (ii) apply a steady-state subsidy (funded by lump-sum taxation applied to all households) to production to eliminate the distortion due to monopolistic competition, and (iii) assume that the policymaker cares only about per capita variables and has no interest in inequality. It is important to note that we still have a heterogeneous agent economy where household income declines with age, and households may be hit by idiosyncratic shocks. As a result, households still attempt to save for both retirement and precautionary reasons, reducing interest rates below their rate of time preference. This situation is described in Figure 2. The black solid line captures the impact of the technology shock on a heterogeneous agent economy of this kind (Hank I), and the light blue dashed line represents the same economy but without any heterogeneity (Rank I). Here, we retain the divine coincidence, and the policymaker reduces interest rates to increase demand in line with supply as the positive productivity shocks increases output. This is optimally achieved without generating any inflation.

Within the HANK I economy, the shock has reduced inequality. There are various drivers of inequality in the face of this shock, some positive, some negative. First, the income of those working has risen relative to retirees, worsening intergenerational inequality. Second, the income of those experiencing a positive idiosyncratic shock has risen relative to those experiencing a negative idiosyncratic shock, again worsening inequality. Third, the fall in interest rates has led to an increase in bond prices, benefiting the holders of such assets. In this version of the economy without fiscal policy, the average steady-state level of assets for each age group is zero, with a variance of $s\eta^2\sigma^2$ where s is the age of the cohort.[8] That is, as an individual ages they are cumulatively hit by positive and negative idiosyncratic shocks, which may lead to them holding positive or negative stocks of assets. Although households wish to accumulate assets for retirement, the net supply of such assets is zero, implying that equilibrium interest rates are lower and consumption falls with age. The positive technology shock leads to a increase in bond prices, which widens the distribution of wealth.[9] Fourth, the reduction in interest rates reduces the marginal propensity to consume μ_t out of adjusted wealth, $m_t^s(i) = A_t^s(i) - \varphi_t \Theta_t^s + \eta_t \left(\xi_t^s(i) - \bar{\xi}\right)$. This effect dominates all the other factors considered above, leading to an overall reduction in consumption inequality.

The remaining two lines in Figure 2 replicate the shock for the same economies but without the steady-state production subsidy. The blue dotted line represents Hank II, the heterogenous agent economy described above but without the subsidy, and the red dot-dash line represents the same economy with the heterogeneity removed, Rank II. Now, the divine coincidence breaks down, and the policymaker does not cut interest rates enough to prevent an initial fall in inflation. As the shock passes, interest rates remain low, leading to a period of inflation rising above target. This pattern of higher inflation in the future helps reduce the initial deflation through its impact on inflation expectations in the New Keynesian Phillips Curve (NKPC) and is a standard feature of policy under commitment in New Keynesian models. Again, the fall in interest rates is less pronounced in the HANK II economy than in the RANK II economy. Since the reduction in interest rates is less significant than in the variants where the steady state has been rendered

[8] See (71)-(72) in Appendix F.

[9] It is important to note that since policy is 'timeless', the policy maker does not try to engineer favorable redistributions of wealth through unexpected policy changes. There is a temptation to do so not only when shocks hit, but in every single period. The policy maker commits not to do so, such that this initial deterioration in the wealth distribution is taken as given by the policy maker. They will, however, seek to adjust policy to reduce inequality thereafter, but without initiating any surprises.

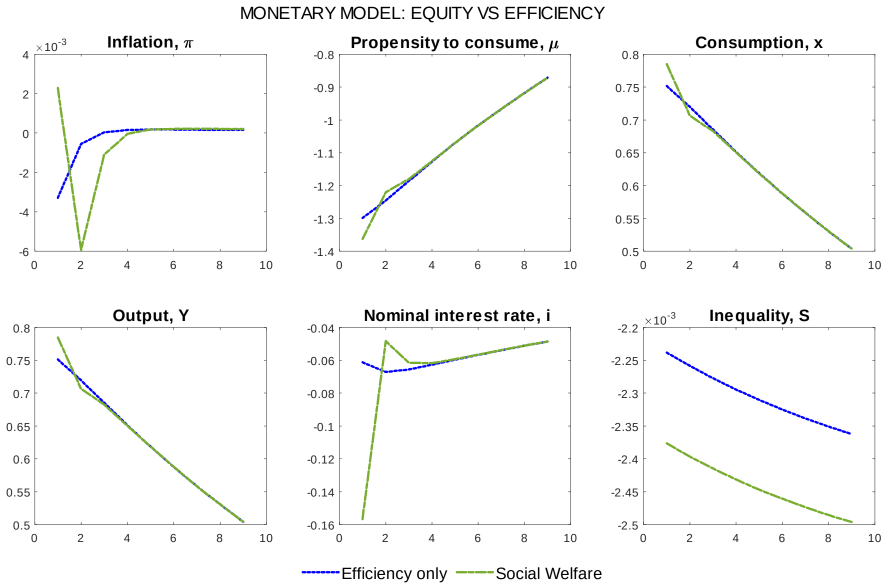

Figure 3: Dynamic responses to a positive technology shock in a Monetary Model. 'Social welfare' scenario assumes $\xi = 1$, while $\xi = 0$ in 'Efficiency only' scenario. In all scenarios we assume HANK model with $\sigma > 0$ and assume declining labour income with $\varkappa > 0$.

efficient through a subsidy, consumption inequality is not reduced by as much, as the reduction in the marginal propensity to consume is not as great.

Having considered the familiar benchmark of economies which exhibit the divine coincidence, we move away from this benchmark in Figure 3 by considering an economy without any subsidy to production and where the policymaker may either care about efficiency only (blue dotted line) or social welfare (green dash-dotted line). The blue dotted line of Figure 3 is the same as the blue dotted line of Figure **??** – the heterogenous agent economy without a subsidy and where the policymaker only cares about efficiency, not equity. In this economy the policy maker fails to reduce interest rates sufficiently to eliminate the fall in inflation, and inequality falls for the reasons discussed above. When we extend the policymaker's remit to include a concern for inequality in line with the social welfare function, the policymaker cuts interest rates more aggressively, reducing the marginal propensity to consume and thereby reducing consumption inequality. This actually raises inflation initially in the face of a positive technology shock, whereas before inflation fell.

Turning to Fig 4, we reintroduce fiscal policy, again considering two different policy objective – social welfare (light blue dashed line) and efficiency only (black line). Fiscal policy can affect households' ability to respond to shocks in various ways: (i) taxation reduces the variance in post-tax income relative to pre-tax income, and (ii) current and expected future taxation also impacts the post-tax earnings households have to borrow against to maintain consumption in the face of shocks. Taxes also discourage worker effort, which can affect inflation through variations in marginal costs. Additionally, government debt provides a vehicle for households to save for

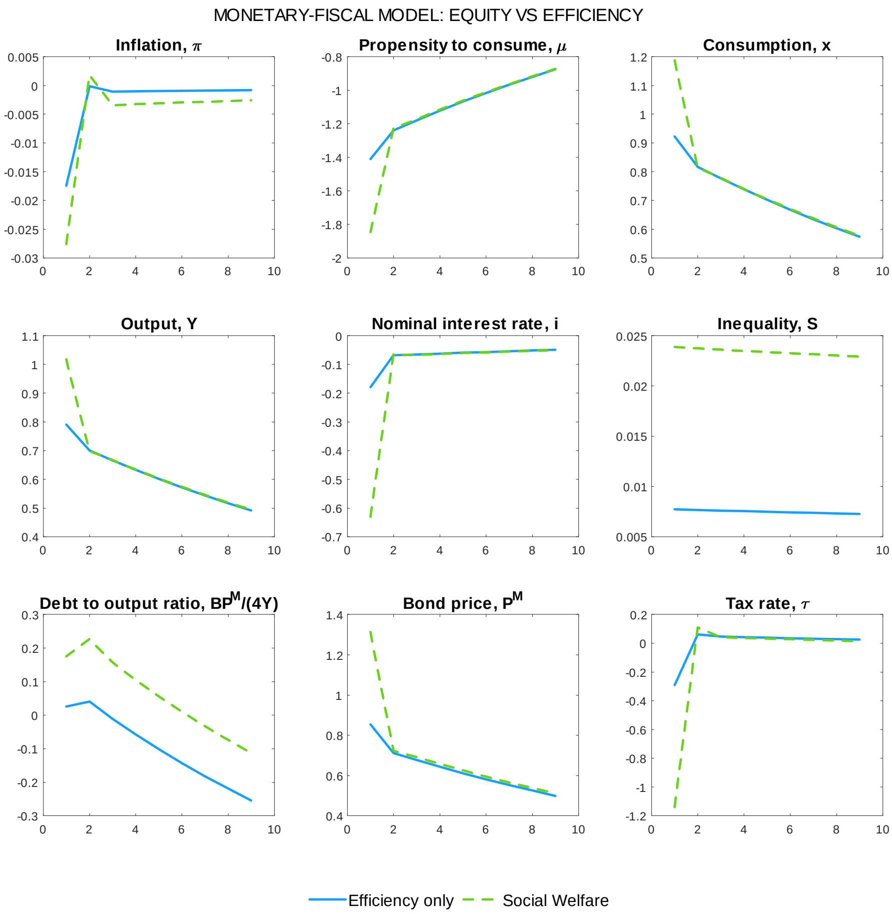

Figure 4: Dynamic responses to a positive technology shock in a Monetary-Fiscal Model. 'Social welfare' scenario assumes $\xi = 1$, while $\xi = 0$ in 'Efficiency only' scenario. In all scenarios we assume HANK model with $\sigma > 0$ and assume declining labour income with $\varkappa > 0$.

retirement and influences real interest rates, which again affects their ability to borrow in the face of negative idiosyncratic shocks. We have already seen that these impacts lead to lower consumption inequality in the steady state in the presence of fiscal policy, particularly when inequality is an element of social welfare. We now consider how having access to fiscal policy changes the policy response to shocks.

The steady-state holdings of assets across age cohorts are quite different in an economy with optimal fiscal policy. Asset holdings (and their variance) rise with age in all cases, and the mean level of assets for a given age is greatest when the policymaker cares about social welfare and lowest when they care about efficiency only, see formulas (71)-(72) in Appendix F. This is because the Ramsey policymaker issues debt to help households save for retirement, although not enough for steady-state interest rates to rise to the golden rule level. As a result, the initial distribution of wealth is significantly impacted by the rise in bond prices caused by the fall in interest rates, which worsens the initial jump in consumption inequality, especially when policy cares about equity as well as efficiency (since initial asset holdings are larger in this case). It is important to stress that under our timelessly optimal policy, policymakers take these revaluation effects as given and commit to not introduce policy surprises in an attempt to engineer revaluations that are favorable to reducing consumption inequality. As a result, we see a jump in inequality, which is greatest when the policymaker's objective is to maximize social welfare, including a desire to mitigate inequality.

The main difference relative to the monetary economy is that the two policies (monetary and fiscal) work together to mitigate inflation and enhance the households' ability to insulate themselves from idiosyncratic shocks. Therefore interest rates and taxation are cut on impact, allowing output to rise in the initial period, and, when the policy objective is to maximize social welfare, almost fully in line with the productivity shock. This is achieved with more deflation than previously, but with a larger decline in the marginal propensity to consume. This is achievable since cutting tax rates reduces marginal costs and leads to falling prices. Therefore a combined tax and interest rate cut can significantly reduce the marginal propensity to consume, μ_t, allowing households to respond to idiosyncratic shocks by more than before, but this no longer leads to a rise in inflation. Beyond the initial period, monetary policy is essentially the same whether or not the policymaker has a concern for equity. However, fiscal policy remains different across these two scenarios, reflecting its impact on inequality. In the case of a concern for equity as well as efficiency, the large initial tax cut is followed by a modest tax increase for a period, which serves to slow the rise in the government debt (although at a higher level than would be chosen by an efficiency-minded policymaker). Thereafter, taxes are slightly lower in the social welfare case, as a lower tax rate enhances households' ability to respond to negative idiosyncratic shocks.[10]

[10] Throughout these simulations we assumed that the variance of idiosyncratic shocks was pro-cyclical. However,

7 Conclusions

Given the growing interest in the distributional consequences of macroeconomic policy, we analyzed a tractable heterogeneous agent OLG economy where households wish to save both for retirement and to insulate themselves from idiosyncratic shocks. Within this economy, we have a well-articulated fiscal policy featuring long-term debt and distortionary taxation as fiscal instruments. This environment allows us to explore the trade-offs between efficiency and equity, both within and across generations, highlighting a number of interesting benchmarks.

We find that, in the steady state, the policymaker weighs inequality concerns heavily, implying that the Ramsey policymaker achieves outcomes much closer to those that would be achieved by a policymaker focused solely on equity, rather than the efficiency concerns implicit in representative agent economies. There are two dimensions to the policymaker's concern with equity: intergenerational equity, driven by the households' gradual withdrawal from the labor market, and intragenerational equity, driven by the idiosyncratic shocks affecting households. Intergenerational equity requires the policymaker to correct the externality in savings behavior implicit in our OLG economy – households wish to save for retirement but do not internalize the impact their savings behavior has on the real interest rate. In the absence of fiscal policy, this would drive interest rates below the households' rate of time preference and imply a decline in consumption as households enter their (phased) retirement. To counteract this, the policymaker issues debt, raising interest rates closer to the households' rate of time preference. If the policymaker were to achieve the 'golden rule' level, debt levels would ensure $R = \beta^{-1}$ and, absent idiosyncratic shocks, households would save sufficient assets to maintain constant consumption throughout their lives, thereby eliminating intergenerational inequality. However, even when concerned solely with equity, the policymaker falls short of this benchmark because raising interest rates through higher debt exacerbates the other dimension of inequality – intragenerational inequality. Specifically, higher interest rates make it more expensive for households to borrow to smooth consumption in the face of negative idiosyncratic shocks. Thus, this second element of inequality leads the policymaker to issue debt at levels that ensure $R < \beta^{-1}$, implying that households do not have access to sufficient assets to maintain consumption throughout their lives.

When we consider that policy also cares about efficiency, there is an additional reason to reduce debt due to the costs of servicing that debt when taxes are distortionary. However, this motivation only reduces the debt-to-GDP ratio from 58% to 53%, as inequality remains the

this did not have a material impact on the results, Results assuming acylical or counter-cyclical variance are available upon request.

primary concern of the policy maker. (If the policy maker was concerned with efficiency only, they would reduce debt much more, to 31% of GDP.)

Turning to the optimal policy response to shocks, an obvious benchmark is the 'divine coincidence' of Blanchard and Galí (2007), where policy optimally responds to variations in the natural level of output without generating any inflation. Our heterogeneous agent OLG economy only supports this result under very special circumstances, beyond the usual requirement that steady-state output is efficient. These include the absence of fiscal policy and a lack of concern for equity on the part of the policymaker. Reintroducing these concerns, we find that optimal monetary policy would relax interest rates more in response to positive technology shocks, as this helps households protect themselves from idiosyncratic shocks. When considering fiscal policy, this implies higher steady-state levels and greater dispersion of assets across households. As a result, shocks have a larger impact on the initial distribution of wealth due to inflation and bond price revaluation effects. In terms of the policy response to shocks, the burden of loosening policy is now shared between monetary policy and fiscal policy, in the form of tax cuts. Tax cuts are particularly effective as they offset the rise in inflation which would be caused by interest rate cuts alone, while both policies facilitate household borrowing in the face of negative idiosyncratic shocks.

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Online Appendix to:

Equity versus Efficiency: Optimal Monetary and Fiscal Policy in a HANK Economy

by

Vasileios Karaferis, Tatiana Kirsanova, and Campbell Leith

A Proof of Proposition 1

Proof. We form the following Lagrangian

$$
\begin{aligned}
L_s = \mathbb{E}_i \sum_{t=s}^{\infty} (\beta\vartheta)^{t-s} \Big(& -\frac{1}{\gamma} e^{-\gamma(c_t^s(i)+\chi G_t)} - \rho e^{\frac{1}{\rho}(l_t^s(i)+\Theta_t^s-\xi_t^s(i))} \\
& + \lambda_t^s(i) \left(\left(c_t^s(i) - \eta_t l_t^s(i) - d_t + T_t + \tilde{P}_t^M a_{t+1}^{L,s}(i) + \tilde{q}_t a_{t+1}^{S,s}(i) \right) (1+\pi_t) \right. \\
& \left. - \left(1 + \varrho \tilde{P}_t^M \right) a_t^{L,s}(i) - a_t^{S,s}(i) \right) \Big)
\end{aligned}
$$

so the FOCs are

$$
\begin{aligned}
0 &= e^{-\gamma(c_t^s(i)+\chi G_t)} + \lambda_t^s(i)(1+\pi_t) \\
0 &= -e^{\frac{1}{\rho}(l_t^s(i)+\Theta_t^s-\xi_t^s(i))} - \lambda_t^s(i)\eta_t(1+\pi_t) \\
0 &= \lambda_t^s(i)\tilde{P}_t^M(1+\pi_t) - \mathbb{E}_i\left(1+\varrho\tilde{P}_{t+1}^M\right)\beta\vartheta\lambda_{t+1}^s(i) \\
0 &= \lambda_t^s(i)\tilde{q}_t(1+\pi_t) - \beta\vartheta\mathbb{E}_i\lambda_{t+1}^s(i)
\end{aligned}
$$

from where (given there is no aggregate risk)

$$
\begin{aligned}
\lambda_t^s(i) &= -\frac{1}{(1+\pi_t)} e^{-\gamma(c_t^s(i)+\chi G_t)} \\
l_t^s(i) &= \rho\log\eta_t - \gamma\rho\left(c_t^s(i)+\chi G_t\right) - \Theta_t^s + \xi_t^s(i) \\
c_t^s(i) &= -\frac{1}{\gamma}\log\frac{\beta\vartheta}{\tilde{q}_t(1+\pi_{t+1})} + \chi G_{t+1} - \chi G_t - \frac{1}{\gamma}\log\mathbb{E}_i e^{-\gamma c_{t+1}^s(i)} \\
\frac{1}{\tilde{q}_t} &= \frac{\left(1+\varrho\tilde{P}_{t+1}^M\right)}{\tilde{P}_t^M}
\end{aligned}
$$

The Euler equation, using normality of consumption distribution, can also be written as

$$
c_t^s(i) = -\frac{1}{\gamma}\log\left(\frac{\beta\vartheta}{\tilde{q}_t(1+\pi_{t+1})}\right) + \chi G_{t+1} - \chi G_t + \mathbb{E}_i c_{t+1}^s(i) - \frac{\gamma}{2}\mathbb{V}_i c_{t+1}^s(i). \tag{44}
$$

To obtain expressions for expectation and variance of consumption, we undertake the following three steps. First, substitute labour supply into the budget constraint,

$$A_{t+1}^{s}(i) = \frac{R_t}{\vartheta}\left(A_t^s(i) + X_t - \eta_t\Theta_t^s + \eta_t\left(\xi_t^s(i) - \bar{\xi}\right) - (1+\rho\gamma\eta_t)\,c_t^s(i)\right) \tag{45}$$

where we denote

$$X_t = \eta_t\left(\rho\log\eta_t + \bar{\xi} - \rho\gamma\chi G_t\right) + d_t - T_t.$$

Second, assume that individual consumption can be parameterized as

$$c_t^s(i) = \mathcal{X}_t + \mu_t\left(A_t^s(i) + \eta_t\left(\xi_t^s(i) - \bar{\xi}\right) - \varphi_t\Theta_t^s\right) \tag{46}$$

and lead one period,

$$\begin{aligned} c_{t+1}^s(i) &= \mathcal{X}_{t+1} + \mu_{t+1}\left(A_{t+1}^s(i) + \eta_{t+1}\left(\xi_{t+1}^s(i) - \bar{\xi}\right) - \varphi_{t+1}\Theta_{t+1}^s\right) \\ &= \mu_{t+1}\left(\frac{R_t}{\vartheta}\left(\begin{array}{c} (1-(1+\rho\gamma\eta_t)\,\mu_t)\left(A_t^s(i) + \eta_t\left(\xi_t^s(i)-\bar{\xi}\right)\right) + X_t \\ -(1+\rho\gamma\eta_t)\,\mathcal{X}_t + (-\eta_t + (1+\rho\gamma\eta_t)\,\mu_t\varphi_t)\,\Theta_t^s \end{array}\right)\right) \\ &\quad + \mathcal{X}_{t+1} + \mu_{t+1}\eta_{t+1}\left(\xi_{t+1}^s(i) - \bar{\xi}\right) - \mu_{t+1}\varphi_{t+1}\left(\Theta_t^s + \varkappa\right) \end{aligned} \tag{47}$$

where in the second line we used the budget constraint, parameterization (46) and the fact that $\Theta_{t+1}^s = \varkappa(t+1-s) = \varkappa(t-s) + \varkappa = \Theta_t^s + \varkappa$.

Finally, we obtain expressions for the expectation and variance terms. Since $c_{t+1}^s(i)$ is normally distributed by i, its mean and variance are determined as follows,

$$\begin{aligned} \mathbb{E}_i c_{t+1}^s(i) &= \mathcal{X}_{t+1} + \mu_{t+1}\left(\begin{array}{c} \frac{R_t}{\vartheta}(1-(1+\rho\gamma\eta_t)\,\mu_t)\left(A_t^s(i) + \eta_t\left(\xi_t^s(i)-\bar{\xi}\right)\right) \\ +\frac{R_t}{\vartheta}\left(X_t - (1+\rho\gamma\eta_t)\,\mathcal{X}_t\right) \\ +\frac{R_t}{\vartheta}\left(-\eta_t + (1+\rho\gamma\eta_t)\,\mu_t\varphi_t\right)\Theta_t^s \end{array}\right) \\ &\quad - \mu_{t+1}\varphi_{t+1}\left(\Theta_t^s + \varkappa\right) \\ \mathbb{V}_i c_{t+1}^s(i) &= \mu_{t+1}^2\eta_{t+1}^2\sigma_{t+1}^2 \end{aligned}$$

(note that $\mathbb{E}_i\xi_{t+1}^s(i) = \bar{\xi}$, but $\mathbb{E}_i\xi_t^s(i) = \xi_t^s(i)$,and $\mathbb{V}_i\xi_{t+1}^s(i) = \sigma_{t+1}^2$, but $\mathbb{V}_i\xi_t^s(i) = 0$).

We now take these expressions and the parameterization (46) and substitute them into the consumption Euler equation (44) to find coefficients $\mathcal{X}_t, \mu_t$ and φ_t. Substitution into the Euler equation yields,

$$\begin{aligned} &\mathcal{X}_t + \mu_t\left(A_t^s(i) + \eta_t^s\left(\xi_t^s(i) - \bar{\xi}\right) - \varphi_t\Theta_t^s\right) \\ &= -\frac{1}{\gamma}\log(\beta R_t) + \chi G_{t+1} - \chi G_t - \mu_{t+1}\varphi_{t+1}\left(\Theta_t^s + \varkappa\right) - \frac{\gamma}{2}\mu_{t+1}^2\eta_{t+1}^2\sigma_{t+1}^2 \\ &+ \mathcal{X}_{t+1} + \mu_{t+1}\frac{R_t}{\vartheta}\left(\begin{array}{c} (1-(1+\rho\gamma\eta_t^s)\,\mu_t)\left(A_t^s(i) + \eta_t^s\left(\xi_t^s(i)-\bar{\xi}\right)\right) \\ +\left(X_t - (1+\rho\gamma\eta_t)\,\mathcal{X}_t\right) + \left(-\eta_t + (1+\rho\gamma\eta_t)\,\mu_t\varphi_t\right)\Theta_t^s \end{array}\right). \end{aligned}$$

Collecting coefficients on independent states, $1, A_t^s(i), \xi_t^s(i), \Theta_t^s$, yields three independent equations on μ_t, κ_t and $\mathcal{X}_t$,

$$\begin{aligned}
\mathcal{X}_t - \mu_t\eta_t\bar{\xi} = {} & -\frac{1}{\gamma}\log(\beta R_t) + \chi\tilde{G}_{t+1} - \chi\tilde{G}_t + \mathcal{X}_{t+1} - \frac{\gamma}{2}\mu_{t+1}^2\eta_{t+1}^2\sigma_{t+1}^2 \\
& + \mu_{t+1}\left(\frac{R_t}{\vartheta}\left(X_t - (1+\rho\gamma\eta_t)\mathcal{X}_t\right) - \frac{R_t}{\vartheta}\left(1-(1+\rho\gamma\eta_t)\mu_t\right)\eta_t\bar{\xi}\right) \\
& - \mu_{t+1}\varphi_{t+1}\varkappa
\end{aligned} \tag{48}$$

$$\mu_t = \mu_{t+1}\frac{R_t}{\vartheta}\left(1-(1+\rho\gamma\eta_t)\mu_t\right) \tag{49}$$

$$-\mu_t\varphi_t = \mu_{t+1}\left(\frac{R_t}{\vartheta}\left(-\eta_t + (1+\rho\gamma\eta_t)\mu_t\varphi_t\right)\right) - \mu_{t+1}\varphi_{t+1} \tag{50}$$

Provided that $\mu_t \neq 0$ the dynamic equation on evolution of the marginal propensity to consume out of adjusted wealth can be expressed as,

$$\frac{1}{\mu_t} - (1+\rho\gamma\eta_t) = \frac{\vartheta}{R_t\mu_{t+1}} \tag{51}$$

the equation for φ_t, the human wealth associated with unit of labor supply, becomes

$$\varphi_t = \eta_t + \frac{\vartheta}{R_t}\varphi_{t+1} \tag{52}$$

and the evolution of the measure of aggregate consumption $\mathcal{X}_t$ is,

$$\begin{aligned}
\mathcal{X}_t = {} & -\frac{\vartheta\mu_t}{\gamma\mu_{t+1}R_t}\log(\beta R_t) + \frac{\vartheta\mu_t}{\mu_{t+1}R_t}\mathcal{X}_{t+1} + \frac{\vartheta\mu_t}{\mu_{t+1}R_t}\chi G_{t+1} \\
& - \frac{\vartheta\mu_t}{\mu_{t+1}R_t}\chi G_t + \mu_t X_t - \frac{\vartheta\mu_t}{R_t}\varkappa\varphi_{t+1} - \frac{\vartheta\mu_t}{\mu_{t+1}R_t}\frac{\gamma}{2}\mu_{t+1}^2\eta_{t+1}^2\sigma_{t+1}^2
\end{aligned}$$

Introducing a new variable,

$$\mathcal{C}_t = \mathcal{X}_t + \chi G_t$$

we arrive at

$$\begin{aligned}
\mathcal{C}_t = {} & -\frac{\vartheta\mu_t}{\gamma\mu_{t+1}R_t}\log(\beta R_t) + \frac{\vartheta\mu_t}{\mu_{t+1}R_t}\mathcal{C}_{t+1} - \frac{\vartheta\mu_t}{R_t}\varkappa\varphi_{t+1} - \frac{\vartheta\mu_t}{\mu_{t+1}R_t}\frac{\gamma}{2}\mu_{t+1}^2\eta_{t+1}^2\sigma_{t+1}^2 \\
& + \mu_t\left(\eta_t\left(\rho\log(\eta_t) + \bar{\xi}\right) + d_t - T_t + \chi G_t\right)
\end{aligned}$$

after all terms in G_t are combined. ■

B Aggregation

Define aggregate consumption, income and labor as,

$$c_t := (1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t-s} \int_0^1 c_t^s (i)\, di$$

$$y_t := (1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t-s} \int_0^1 y_t^s (i)\, di$$

$$n_t := \int_0^1 n_t (j)\, dj = (1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t-s} \int_0^1 l_t^s (i)\, di$$

and aggregate actuarial bonds, $J = \{S, L\}$,

$$\vartheta a_t^J := (1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t-s} \int_0^1 a_t^{J,s} (i)\, di.$$

To aggregate the household budget constraint, we need to compute $(1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t-s} \int_0^1 a_{t+1}^{J,s} (i)\, di$. Note that

$$\begin{aligned}\vartheta a_{t+1}^J &= (1-\vartheta) \sum_{s=-\infty}^{t+1} \vartheta^{t+1-s} \int_0^1 a_{t+1}^{J,s} (i)\, di = (1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t+1-s} \int_0^1 a_{t+1}^{J,s} (i)\, di \\ &+ (1-\vartheta) \int_0^1 a_{t+1}^{J,t+1} (i)\, di \\ &= \vartheta (1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t-s} \int_0^1 a_{t+1}^{J,s} (i)\, di\end{aligned}$$

then

$$a_{t+1}^J = (1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t-s} \int_0^1 a_{t+1}^{J,s} (i)\, di$$

It follows that,

$$\vartheta A_t = (1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t-s} \int_0^1 A_t^s (i)\, di,$$

$$A_{t+1}^s = (1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t-s} \int_0^1 A_{t+1}^s (i)\, di,$$

Finally, note that

$$\begin{aligned}\sum_{s=-\infty}^{t} \vartheta^{t-s} (t-s) &= \vartheta^{t-t} (t-t) + \vartheta^{t-t+1} (t-t+1) + \vartheta^{t-t+2} (t-2) = ... \\ &= \sum_{k=1}^{\infty} k \vartheta^k = \frac{\vartheta}{(1-\vartheta)^2}\end{aligned}$$

so that aggregation of lost retirement hours across the whole population yields

$$
\begin{aligned}
(1-\vartheta)\sum_{s=-\infty}^{t}\vartheta^{t-s}\int_0^1 \Theta_t^s di &= (1-\vartheta)\sum_{s=-\infty}^{t}\vartheta^{t-s}\int_0^1 \varkappa\,(t-s)\,di \\
&= \varkappa\,(1-\vartheta)\sum_{s=-\infty}^{t}\vartheta^{t-s}\,(t-s) \\
&= \frac{\varkappa\vartheta}{1-\vartheta}
\end{aligned}
$$

Aggregation of the household budget constraint (2) yields,

$$
\frac{\vartheta}{R_t}\frac{\left(\frac{\tilde{P}_t^M}{\tilde{q}_t}a_{t+1}^L + a_{t+1}^S\right)}{(1+\pi_{t+1})} = \vartheta\frac{\left(\left(1+\varrho\tilde{P}_t^M\right)a_t^L + a_t^S\right)}{(1+\pi_t)} + y_t - c_t \tag{53}
$$

or

$$
\frac{\vartheta}{R_t}A_{t+1} = \vartheta A_t + y_t - c_t
$$

where

$$
A_t = \frac{\left(\left(1+\varrho\tilde{P}_t^M\right)a_t^L + a_t^S\right)}{(1+\pi_t)} = \frac{\left(\frac{\tilde{P}_{t-1}^M}{\tilde{q}_{t-1}}a_t^L + a_t^S\right)}{(1+\pi_t)}
$$

and

$$
y_t = \eta_t n_t + d_t - T_t.
$$

C Proof of Proposition 2.

Proof. We start with the derived relationship,

$$
\begin{aligned}
\mathcal{X}_t = &-\frac{\vartheta\mu_t}{\gamma\mu_{t+1}R_t}\log(\beta R_t) + \frac{\vartheta\mu_t}{\mu_{t+1}R_t}\mathcal{X}_{t+1} + \frac{\vartheta\mu_t}{\mu_{t+1}R_t}\chi G_{t+1} \\
&-\frac{\vartheta\mu_t}{\mu_{t+1}R_t}\chi G_t + \mu_t X_t - \frac{\vartheta\mu_t}{R_t}\varkappa\varphi_{t+1} - \frac{\vartheta\mu_t}{\mu_{t+1}R_t}\frac{\gamma}{2}\mu_{t+1}^2\eta_{t+1}^2\sigma_{t+1}^2
\end{aligned}
$$

Recall that

$$
\mathcal{X}_t = \mathcal{C}_t - \chi G_t
$$

and

$$
c_t = \mathcal{C}_t - \chi G_t + \mu_t\left(\vartheta A_t - \frac{\vartheta\varkappa}{(1-\vartheta)}\varphi_t\right)
$$

So we can parameterize

$$\mathcal{X}_t = \mathcal{C}_t - \chi G_t = c_t - \mu_t \left(\vartheta A_t - \frac{\vartheta \varkappa}{(1-\vartheta)} \varphi_t \right)$$
$$\mathcal{X}_{t+1} = c_{t+1} - \mu_{t+1} \left(\vartheta A_{t+1} - \frac{\vartheta \varkappa}{(1-\vartheta)} \varphi_{t+1} \right)$$

and combine these two relationships

$$\begin{aligned}
& c_t - \mu_t \left(\vartheta A_t - \frac{\vartheta \varkappa}{(1-\vartheta)} \varphi_t \right) \\
& = -\frac{\vartheta \mu_t}{\gamma \mu_{t+1} R_t} \log(\beta R_t) + \frac{\vartheta \mu_t}{\mu_{t+1} R_t} \left(c_{t+1} - \mu_{t+1} \left(\vartheta A_{t+1} - \frac{\vartheta \varkappa}{(1-\vartheta)} \varphi_{t+1} \right) \right) \\
& + \frac{\vartheta \mu_t}{\mu_{t+1} R_t} \chi G_{t+1} - \frac{\vartheta \mu_t}{\mu_{t+1} R_t} \chi G_t + \mu_t X_t - \frac{\vartheta \mu_t}{R_t} \varkappa \varphi_{t+1} - \frac{\vartheta \mu_t}{\mu_{t+1} R_t} \frac{\gamma}{2} \mu_{t+1}^2 \eta_{t+1}^2 \sigma_{t+1}^2
\end{aligned}$$

Substitute

$$X_t = \eta_t \left(\rho \log(\eta_t) + \bar{\xi} - \rho \gamma \chi G_t \right) + d_t - T_t$$

and use the budget constraint

$$\vartheta A_t = \frac{\vartheta}{R_t} A_{t+1} - \eta_t \left(\rho \log(\eta_t) + \bar{\xi} \right) + \frac{\varkappa \vartheta}{(1-\vartheta)} \eta_t + \rho \gamma \eta_t \chi G_t + \rho \gamma \eta_t c_t - d_t + T_t + c_t$$

and (6)-(7) to arrive at the following Euler equation

$$c_t + \chi G_t = -\frac{1}{\gamma} \log(\beta R_t) + c_{t+1} + \chi G_{t+1} + (1-\vartheta) \mu_{t+1} A_{t+1} - \frac{\gamma}{2} \mu_{t+1}^2 \eta_{t+1}^2 \sigma_{t+1}^2 - \mu_{t+1} \varkappa \varphi_{t+1}$$

Labor supply (5) is straightforwardly aggregated as,

$$n_t = \rho \log(\eta_t) - \varkappa \frac{\vartheta}{1-\vartheta} - \rho \gamma (c_t + \chi G_t) + \bar{\xi}$$

■

D Derivation of Phillips Curve

Firm j solves the following optimization problem

$$\max_{P_t(j)} \sum_{t=0}^{\infty} \beta^t \left(\left(\frac{P_t(j)}{P_t} Y_t(j) - (1-s) w_t n_t(j) \right) - \frac{\Phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t \right)$$

subject to monopolistic demand

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon_t} Y_t$$

and its production function

$$Y_t(j) = z_t n_t(j)$$

Substitute these constraints into the definition of profits to create an unconstrained problem,

$$\max_{P_t(j)} \sum_{t=0}^{\infty} \beta^t \left(\left(\frac{P_t(j)}{P_t} - (1-s)\frac{w_t}{z_t} \right) \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon_t} Y_t - \frac{\Phi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t \right)$$

which yields the following first order condition,

$$0 = \beta^t \left((1-\varepsilon_t) \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon_t} \frac{Y_t}{P_t} + \varepsilon_t (1-s) \frac{w_t}{z_t} \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon_t - 1} \frac{Y_t}{P_t} - \Phi \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right) \frac{Y_t}{P_{t-1}(j)} \right)$$
$$+ \beta^{t+1} \left(\Phi \left(\frac{P_{t+1}(j)}{P_t(j)} - 1 \right) Y_{t+1} \frac{P_{t+1}(j)}{P_t^2(j)} \right)$$

Consider a symmetrical equilibrium where $P_t(j) = P_t$ and we obtain the New Keynesian Phillips curve,

$$\pi_t(1+\pi_t) = \frac{1 - \varepsilon_t + (1-s)\varepsilon_t \frac{w_t}{z_t}}{\Phi} + \beta \frac{Y_{t+1}}{Y_t} \pi_{t+1}(1+\pi_{t+1})$$

The profits of firms are distributed as dividends,

$$d_t = (Y_t - (1-s) w_t n_t) - \frac{\Phi}{2} \pi_t^2 Y_t$$

E Financial Intermediaries

Financial intermediaries trade actuarial and government bonds. At time t they buy short and long-term actuarial bonds and pay with short and long term government bonds, so the budget constraint of intermediaries is

$$-\tilde{P}_t^M a_{t+1}^L - \tilde{q}_t a_{t+1}^S + P_t^M b_{t+1}^L + q_t b_{t+1}^S \leqslant 0, \tag{54}$$

where $a_{t+1}^J = (1-\vartheta) \sum_{s=-\infty}^{t} \vartheta^{t-s} \int_0^1 a_{t+1}^{J,s}(i)\, di$.

Their profit one period later is, therefore

$$\Pi = \left(1 + \varrho P_{t+1}^M\right) b_{t+1}^L + b_{t+1}^S - \left(1 + \varrho \tilde{P}_{t+1}^M\right) \vartheta a_{t+1}^L - \vartheta a_{t+1}^S$$

where b_{t+1}^J are total government bonds at time $t+1$, and ϑa_{t+1}^J are total actuarial bonds at time $t+1$, i.e. $\vartheta a_{t+1}^J = (1-\vartheta) \sum_{s=-\infty}^{t+1} \vartheta^{t+1-s} \int_0^1 a_{t+1}^{J,s}(i)\, di$.

The Lagrangian is

$$\Pi = \left(1 + \varrho P_{t+1}^M\right) b_{t+1}^L + b_{t+1}^S - \left(1 + \varrho \tilde{P}_{t+1}^M\right) \vartheta a_{t+1}^L - \vartheta a_{t+1}^S$$
$$+ \lambda_t \left(-\tilde{P}_t^M a_{t+1}^L - \tilde{q}_t a_{t+1}^S + P_t^M b_{t+1}^L + q_t b_{t+1}^S \right)$$

and the first order conditions are,

$$\begin{aligned}
\frac{\partial}{\partial b_{t+1}^{L}} &: \left(1+\varrho P_{t+1}^{M}\right)+\lambda_t P_t^{M} \\
\frac{\partial}{\partial b_{t+1}^{S}} &: 1+\lambda_t q_t \\
\frac{\partial}{\partial a_{t+1}^{L}} &: -\left(1+\varrho \tilde{P}_{t+1}^{M}\right)\vartheta-\lambda_t \tilde{P}_t^{M} \\
\frac{\partial}{\partial a_{t+1}^{S}} &: -\vartheta-\lambda_t \tilde{q}_t
\end{aligned}$$

From where we have,

$$\frac{1}{\tilde{q}_t}=\frac{\left(1+\varrho \tilde{P}_{t+1}^{M}\right)}{\tilde{P}_t^{M}} \tag{55}$$

$$\tilde{q}_t=\vartheta q_t \tag{56}$$

$$\frac{1}{q_t}=\frac{\left(1+\varrho P_{t+1}^{M}\right)}{P_t^{M}} \tag{57}$$

and the profit is zero,

$$\begin{aligned}
\Pi &= \left(1+\varrho P_{t+1}^{M}\right) b_{t+1}^{L}+b_{t+1}^{S}-\vartheta a_{t+1}^{S}-\vartheta\left(1+\varrho \tilde{P}_{t+1}^{M}\right) a_{t+1}^{L} \\
&= \frac{1}{q_t}\left(P_t^{M} b_{t+1}^{L}+q_t b_{t+1}^{S}-\tilde{q}_t a_{t+1}^{S}-\tilde{P}_t^{M} a_{t+1}^{L}\right)=0
\end{aligned} \tag{58}$$

and

$$R_t=\frac{\vartheta}{\tilde{q}_t\left(1+\pi_{t+1}\right)}=\frac{1}{q_t\left(1+\pi_{t+1}\right)} \tag{59}$$

F Social Welfare Function

F.1 Aggregation of Welfare

Recall that

$$\begin{aligned}
l_t^{s}(i) &= \rho \log \eta_t-\Theta_t^{s}-\rho\gamma\left(c_t^{s}(i)+\chi G_t\right)+\xi_t^{s}(i) \\
c_t^{s}(i) &= \mathcal{C}_t-\chi G_t+\mu_t m_t^{s}(i) \\
m_t^{s}(i) &= A_t^{s}(i)+\eta_t\left(\xi_t^{s}(i)-\bar{\xi}\right)-\varphi_t \Theta_t^{s}
\end{aligned}$$

so the (remaining at p) life-time utility of an agent born at s at time $p>s$ can be written as (after substituting labor supply)

$$W_p^{s}(i)=\sum_{t=p}^{\infty}(\beta\vartheta)^{t-p}\,U_t^{s}(i) \tag{60}$$

where

$$
\begin{aligned}
U_t^s(i) &= -\frac{1}{\gamma} e^{-\gamma\left(c_t^s(i)+\chi G_t\right)} - \rho e^{\frac{1}{\rho}\left(l_t^s(i)+\Theta_t^s-\xi_t^s(i)\right)} \\
&= -\frac{1}{\gamma} e^{-\gamma\left(c_t^s(i)+\chi G_t\right)} - \rho e^{\frac{1}{\rho}\left(\rho \log(\eta_t)-\rho\gamma\left(c_t^s(i)+\chi G_t\right)\right)} \\
&= -\frac{1}{\gamma} e^{-\gamma\left(c_t^s(i)+\chi G_t\right)} - \rho \eta_t e^{-\gamma\left(c_t^s(i)+\chi G_t\right)} \\
&= -\frac{1}{\gamma}\left(1+\gamma\rho\eta_t\right) e^{-\gamma\left(c_t^s(i)+\chi G_t\right)} \\
&= -\frac{1}{\gamma}\left(1+\gamma\rho\eta_t\right) e^{-\gamma\left(\mathcal{C}_t+\mu_t m_t^s(i)\right)}
\end{aligned}
$$

The social welfare function at time $t=0$ is defined as

$$
\mathbb{W}_0 = (1-\vartheta)\sum_{s=-\infty}^{0}\vartheta^{-s}\int_0^1 W_0^s(i)\,di + \sum_{s=1}^{\infty}(1-\vartheta)\beta^s\int_0^1 W_s^s(i)\,di \tag{61}
$$

where the first term is utility of generations that are alive at time zero. The second term is utility of unborn generations, with $s>0$, each such generation is treated with weight β^s to ensure the social welfare function is time-consistent.

We can rewrite the welfare function in a more convenient way as follows. Denote

$$
\mathcal{U}_t^s = -\frac{1}{\gamma}\left(1+\gamma\rho\eta_t\right)\int_0^1 e^{-\gamma\left(\mathcal{C}_t+\mu_t m_t^s(i)\right)} di
$$

as the t-period utility of a *cohort* born at time s.

Then

$$
\begin{aligned}
\frac{\mathbb{W}_0}{(1-\vartheta)} &= \mathcal{U}_0^0 + \vartheta\mathcal{U}_0^{-1} + \vartheta^2\mathcal{U}_0^{-2} + \ldots \\
&+ \beta\left(\mathcal{U}_0^1 + \vartheta\mathcal{U}_0^0 + \vartheta^2\mathcal{U}_0^{-1} + \ldots\right) + \ldots \\
&+ \beta^t\left(\mathcal{U}_t^t + \vartheta\mathcal{U}_t^{t-1} + \vartheta^2\mathcal{U}_t^{t-2} + \ldots + \vartheta^s\mathcal{U}_t^{t-s}\right) + \ldots \\
&= \sum_{t=0}^{\infty}\beta^t\sum_{s=0}^{\infty}\vartheta^s\mathcal{U}_t^{t-s} = \sum_{t=0}^{\infty}\beta^t\sum_{v=-\infty}^{t}\vartheta^{t-v}\mathcal{U}_t^v
\end{aligned}
$$

where in the last line we used new index $v=t-s$.

Recycling notation, we get

$$
\mathbb{W}_0 = -\frac{1}{\gamma}\sum_{t=0}^{\infty}\beta^t\left(1+\gamma\rho\eta_t\right)e^{-\gamma\mathcal{C}_t}\left((1-\vartheta)\sum_{s=-\infty}^{t}\vartheta^{t-s}\int_0^1 e^{-\gamma\mu_t m_t^s(i)}di\right) \tag{62}
$$

$$
\mathbb{W}_0 = -\frac{1}{\gamma}\sum_{t=0}^{\infty}\beta^t\left(1+\gamma\rho\eta_t\right)e^{-\gamma\mathcal{C}_t}\left((1-\vartheta)\sum_{s=-\infty}^{t}\vartheta^{t-s}\int_0^1 e^{-\gamma\mu_t m_t^s(i)}di\right) \tag{63}
$$

Denote

$$\Sigma_t = (1-\vartheta)\sum_{s=-\infty}^{t}\vartheta^{t-s}\int_0^1 e^{-\gamma\mu_t m_t^s(i)}di$$

so that

$$\mathbb{W}_0 = \sum_{t=0}^{\infty}\beta^t \mathbb{U}_t$$

where

$$\mathbb{U}_t = -\frac{1}{\gamma}\left(1+\gamma\rho\eta_t\right)e^{-\gamma\mathcal{C}_t}\Sigma_t$$

Here $\left(1+\gamma\rho\eta_t\right)e^{-\gamma\mathcal{C}_t}$ only depends on aggregate variables, so will be the same for a representative agent. $\Sigma_t = (1-\vartheta)\sum_{s=-\infty}^{t}\vartheta^{t-s}\int_0^1 e^{-\gamma\mu_t m_t^s(i)}di$ captures the welfare cost of inequality. It is increasing in the within and across cohort dispersion of consumption.

F.2 Recursion

We now derive the Σ_t recursion, which underpins the recursive evolution of inequality, S_t.

$$\begin{aligned}
\Sigma_t &= (1-\vartheta)\sum_{s=-\infty}^{t}\vartheta^{t-s}\int_0^1 e^{-\gamma\mu_t m_t^s(i)}di \\
&= (1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-s}\int_0^1 e^{-\gamma\mu_t m_t^s(i)}di + (1-\vartheta)\int_0^1 e^{-\gamma\mu_t m_t^t(i)}di \\
&= (1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-s}e^{\gamma\mu_t\varkappa(t-s)\varphi_t}\int_0^1 e^{-\gamma\mu_t\left(A_t^s(i)+\eta_t\left(\xi_t^s(i)-\bar{\xi}\right)\right)}di \\
&+ (1-\vartheta)\int_0^1 e^{-\gamma\mu_t\eta_t\left(\xi_t^s(i)-\bar{\xi}\right)}di \\
&= (1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-s}e^{\gamma\mu_t\varkappa(t-s)\varphi_t}I_t + (1-\vartheta)\,e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}
\end{aligned}$$

where

$$I_t = \int_0^1 e^{-\gamma\mu_t\left(A_t^s(i)+\eta_t\left(\xi_t^s(i)-\bar{\xi}\right)\right)}di = \int_0^1 e^{-\gamma\mu_t A_t^s(i)}e^{-\gamma\mu_t\left(\eta_t\left(\xi_t^s(i)-\bar{\xi}\right)\right)}di$$

Integral I_t is an expectation of a product of two functions (uniformly distributed), and as $A_t^s(i)$ is not correlated with $\left(\xi_t^s(i)-\bar{\xi}\right)$, see 1, then the expectation of the product is equal to the product of the expectations, and we can rewrite as

$$I_t = \int_0^1 e^{-\gamma\mu_t\left(\eta_t\left(\xi_t^s(j)-\bar{\xi}\right)\right)}dj\int_0^1 e^{-\gamma\mu_t A_t^s(i)}di = e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}\int_0^1 e^{-\gamma\mu_t A_t^s(i)}di$$

Recall the budget constraint (45),

$$A_{t+1}^{s}(i)=\frac{R_t}{\vartheta}\left(A_t^s(i)+\eta_t\left(\xi_t^s(i)-\bar{\xi}\right)+X_t-\eta_t\Theta_t^s-(1+\rho\gamma\eta_t)\,c_t^s(i)\right)$$

substitute out consumption using (46)

$$A_{t+1}^{s}(i)=\frac{R_t}{\vartheta}\left(\begin{array}{c}(1-(1+\rho\gamma\eta_t)\,\mu_t)\left(A_t^s(i)+\eta_t\left(\xi_t^s(i)-\bar{\xi}\right)\right)\\ +X_t-(1+\rho\gamma\eta_t)\,\mathcal{X}_t-(\eta_t-(1+\rho\gamma\eta_t)\,\mu_t\varphi_t)\,\Theta_t^s\end{array}\right)$$

and simplify using (51) and (50)

$$\mu_{t+1}A_{t+1}^{s}(i)=\left(\begin{array}{c}\mu_t\left(A_t^s(i)+\eta_t\left(\xi_t^s(i)-\bar{\xi}\right)\right)-(\mu_t\varphi_t-\mu_{t+1}\varphi_{t+1})\,\Theta_t^s\\ +\mu_{t+1}\frac{R_t}{\vartheta}\left(X_t-(1+\rho\gamma\eta_t)\,\mathcal{X}_t\right)\end{array}\right).$$

Take a lag and substitute this expression into the formula for I_t to obtain a recursion for this integral,

$$\begin{aligned}
&I_t=\int_0^1 e^{-\gamma\mu_t\left(A_t^s(i)+\eta_t\left(\xi_t^s(i)-\bar{\xi}\right)\right)}di=e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}\int_0^1 e^{-\gamma\mu_t A_t^s(i)}di\\
&=e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}\int_0^1 e^{-\gamma\left(\mu_{t-1}\left(A_{t-1}^s(i)+\eta_{t-1}\left(\xi_{t-1}^s(i)-\bar{\xi}\right)\right)\right)}\\
&\times e^{-\gamma\left(\mu_t\frac{R_{t-1}}{\vartheta}(X_{t-1}-(1+\rho\gamma\eta_{t-1})\mathcal{X}_{t-1})-(\mu_{t-1}\varphi_{t-1}-\mu_t\varphi_t)\Theta_{t-1}^s\right)}di\\
&=e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}e^{-\gamma\left(\mu_t\frac{R_{t-1}}{\vartheta}(X_{t-1}-(1+\rho\gamma\eta_{t-1})\mathcal{X}_{t-1})-(\mu_{t-1}\varphi_{t-1}-\mu_t\varphi_t)\Theta_{t-1}^s\right)}\\
&\times\int_0^1 e^{-\gamma\left(\mu_{t-1}\left(A_{t-1}^s(i)+\eta_{t-1}\left(\xi_{t-1}^s(i)-\bar{\xi}\right)\right)\right)}di\\
&=e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}e^{-\gamma\left(\mu_t\frac{R_{t-1}}{\vartheta}\left(X_{t-1}-\left(1+\tau_{t-1}^c+\rho\gamma\eta_{t-1}\right)\mathcal{X}_{t-1}\right)-\left(\mu_{t-1}\varphi_{t-1}-\mu_t\varphi_t\frac{\varkappa_t}{\varkappa_{t-1}}\right)\varkappa(t-1-s)\right)}I_{t-1}\\
&=e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}e^{-\gamma\left(\mu_t\frac{R_{t-1}}{\vartheta}(X_{t-1}-(1+\rho\gamma\eta_{t-1})\mathcal{X}_{t-1})-(\mu_{t-1}\varphi_{t-1}\varkappa-\mu_t\varphi_t\varkappa)(t-1-s)\right)}I_{t-1}
\end{aligned}$$

Note that, by definition,

$$\begin{aligned}
\Sigma_{t-1}&=(1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-1-s}\int_0^1 e^{-\gamma\mu_{t-1}m_{t-1}^s(i)}di\\
&=(1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-1-s}\int_0^1 e^{-\gamma\mu_{t-1}\left(A_{t-1}^s(i)+\eta_{t-1}\left(\xi_{t-1}^s(i)-\bar{\xi}\right)-\varphi_{t-1}\Theta_{t-1}^s\right)}di\\
&=(1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-1-s}e^{-\gamma\mu_{t-1}\left(-\varphi_{t-1}\Theta_{t-1}^s\right)}\int_0^1 e^{-\gamma\mu_{t-1}\left(A_{t-1}^s(i)+\eta_{t-1}\left(\xi_{t-1}^s(i)-\bar{\xi}\right)\right)}di
\end{aligned}$$

so that

$$\Sigma_{t-1}=(1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-1-s}e^{\gamma\mu_{t-1}\varphi_{t-1}\varkappa(t-1-s)}I_{t-1}$$

Now, isolate this term,

$$
\begin{aligned}
\Sigma_t &= (1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-s}e^{\gamma\mu_t\varkappa(t-s)\varphi_t}I_t + (1-\vartheta)\,e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}\\
&= (1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-s}e^{\gamma\mu_t\varkappa(t-s)\varphi_t}e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}\\
&\times e^{-\gamma\left(\mu_t\frac{R_{t-1}}{\vartheta}(X_{t-1}-(1+\rho\gamma\eta_{t-1})\mathcal{X}_{t-1})-(\mu_{t-1}\varphi_{t-1}\varkappa-\mu_t\varphi_t\varkappa)(t-1-s)\right)}I_{t-1}\\
&+ (1-\vartheta)\,e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}\\
&= e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}\,(1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-s}e^{\gamma\mu_t\varkappa(t-s)\varphi_t+\gamma(\mu_{t-1}\varphi_{t-1}\varkappa-\mu_t\varphi_t\varkappa)(t-1-s)}\\
&\times e^{-\gamma\left(\mu_t\frac{R_{t-1}}{\vartheta}(X_{t-1}-(1+\rho\gamma\eta_{t-1})\mathcal{X}_{t-1})\right)}I_{t-1}\\
&+ (1-\vartheta)\,e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}\\
&= e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}\,(1-\vartheta)\sum_{s=-\infty}^{t-1}\vartheta^{t-s}e^{\gamma(\mu_t\varphi_t\varkappa+\mu_{t-1}\varphi_{t-1}\varkappa(t-1-s))}\\
&\times e^{-\gamma\left(\mu_t\frac{R_{t-1}}{\vartheta}(X_{t-1}-(1+\rho\gamma\eta_{t-1})\mathcal{X}_{t-1})\right)}I_{t-1}\\
&+ (1-\vartheta)\,e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}\\
&= e^{\gamma\mu_t\varphi_t\varkappa}e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}e^{-\gamma\left(\mu_t\frac{R_{t-1}}{\vartheta}(X_{t-1}-(1+\rho\gamma\eta_{t-1})\mathcal{X}_{t-1})\right)}\vartheta\,(1-\vartheta)\\
&\times\sum_{s=-\infty}^{t-1}\vartheta^{t-s-1}e^{\gamma\mu_{t-1}\varphi_{t-1}\varkappa(t-1-s)}I_{t-1} + (1-\vartheta)\,e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}
\end{aligned}
$$

to obtain the recursive relationship,

$$
\Sigma_t = \vartheta e^{-\gamma\mu_t\left(\frac{R_{t-1}}{\vartheta}(X_{t-1}-(1+\rho\gamma\eta_{t-1})\mathcal{X}_{t-1})-\varkappa\varphi_t\right)}e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}\Sigma_{t-1} + (1-\vartheta)\,e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2}
$$

Introduce new variable Z_t to obtain

$$
\Sigma_t = \left(e^{-\frac{\gamma}{\vartheta}\mu_t(R_{t-1}Z_{t-1}-\vartheta\varkappa\varphi_t)}\vartheta\Sigma_{t-1} + 1-\vartheta\right)e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2} \tag{64}
$$

where

$$
\begin{aligned}
Z_t &= X_t - (1+\rho\gamma\eta_t)\,\mathcal{X}_t \qquad (65)\\
&= \eta_t\left(\rho\log(\eta_t)+\bar{\xi}-\rho\gamma\chi G_t\right) + d_t - T_t - (1+\rho\gamma\eta_t)\,(\mathcal{C}_t-\chi G_t)\\
&= \eta_t\left(\rho\log(\eta_t)+\bar{\xi}\right) - (1+\rho\gamma\eta_t)\,\mathcal{C}_t + \chi G_t + d_t - T_t
\end{aligned}
$$

We can represent Z_t in a different form,

$$c_t + \chi G_t - \vartheta\mu_t \left(A_t - \frac{\varkappa}{1-\vartheta}\varphi_t \right) = \mathcal{C}_t$$

then

$$Z_t = \eta_t \left(\rho \log(\eta_t) + \bar{\xi}\right) - (1+\rho\gamma\eta_t) \left(c_t + \chi G_t - \vartheta\mu_t \left(A_t - \frac{\varkappa}{1-\vartheta}\varphi_t \right) \right) + \chi G_t + d_t - T_t$$

use

$$y_t = \eta_t \rho \log(\eta_t) + \eta_t \bar{\xi} - \eta_t \varkappa \frac{\vartheta}{1-\vartheta} - \rho\gamma\eta_t c_t - \rho\gamma\chi\eta_t G_t + d_t - T_t.$$

to obtain

$$y_t + \varkappa \frac{\vartheta}{(1-\vartheta)}\eta_t + \rho\gamma\eta_t c_t - d_t + T_t + \eta_t\rho\gamma\chi G_t = \eta_t \left(\rho \log(\eta_t) + \bar{\xi}\right).$$

$$Z_t = y_t - c_t + (1+\rho\gamma\eta_t)\mu_t\vartheta A_t + \frac{\vartheta}{(1-\vartheta)}\varkappa\eta_t - (1+\rho\gamma\eta_t)\mu_t \frac{\vartheta\varkappa}{1-\vartheta}\varphi_t \tag{66}$$

$$(1+\rho\gamma\eta_t)\mu_t = 1 - \frac{\vartheta\mu_t}{\mu_{t+1}R_t}$$

$$Z_t = y_t - c_t + (1+\rho\gamma\eta_t)\mu_t\vartheta A_t - \frac{\vartheta}{(1-\vartheta)} \frac{\vartheta}{\mu_{t+1}R_t} \left(\varkappa\mu_{t+1}\varphi_{t+1} - \varkappa\mu_t\varphi_t\right)$$

Furthermore, the aggregated budget constraint,

$$\frac{\vartheta}{R_t} A_{t+1} - \vartheta A_t = y_t - c_t$$

using,

$$\begin{aligned}
Z_t &= \frac{\vartheta}{R_t} A_{t+1} - \vartheta A_t + (1+\rho\gamma\eta_t)\mu_t\vartheta A_t - \frac{\vartheta}{(1-\vartheta)} \frac{\vartheta}{\mu_{t+1}R_t} \left(\varkappa\mu_{t+1}\varphi_{t+1} - \varkappa\mu_t\varphi_t\right) \\
&= \frac{\vartheta}{R_t} A_{t+1} - \frac{\vartheta\mu_t}{\mu_{t+1}R_t}\vartheta A_t - \frac{\vartheta}{(1-\vartheta)} \frac{\vartheta}{\mu_{t+1}R_t} \left(\varkappa\mu_{t+1}\varphi_{t+1} - \varkappa\mu_t\varphi_t\right) \\
&= \frac{\vartheta}{R_t} \left(A_{t+1} - \frac{\vartheta}{(1-\vartheta)}\varkappa\varphi_{t+1} \right) - \frac{\vartheta\mu_t}{\mu_{t+1}R_t} \left(\vartheta A_t - \frac{\vartheta}{(1-\vartheta)}\varkappa\varphi_t \right)
\end{aligned}$$

implies,

$$Z_t = \frac{\vartheta}{\mu_{t+1}R_t} \left(\mu_{t+1} \left(A_{t+1} - \frac{\vartheta}{(1-\vartheta)}\varkappa\varphi_{t+1} \right) - \mu_t \left(\vartheta A_t - \frac{\vartheta}{(1-\vartheta)}\varkappa\varphi_t \right) \right) \tag{67}$$

It is apparent that if the aggregate asset holding is zero then $Z_t = 0$ and we obtain the same recursive formula for Σ_t as reported in Acharya et al (2020).

Using the intermediation constraint (20) we rewrite (67) as,

$$\mu_{t+1}R_tZ_t = \mu_{t+1}\left(B_{t+1} - \vartheta\frac{\vartheta}{(1-\vartheta)}\varkappa\varphi_{t+1}\right) - \vartheta\mu_t\left(B_t - \frac{\vartheta}{(1-\vartheta)}\varkappa\varphi_t\right) \tag{68}$$

Introducing another new variable

$$W_t = \mu_t\left(B_t - \frac{\vartheta\varkappa}{(1-\vartheta)}\varphi_t\right)$$

then

$$\mu_{t+1}R_tZ_t = W_{t+1} - \vartheta W_t + \vartheta\varkappa\mu_{t+1}\varphi_{t+1} \tag{69}$$

Denote

$$S_t = e^{\gamma\mu_t\left(B_t - \frac{\vartheta\varkappa}{(1-\vartheta)}\varphi_t\right)}\Sigma_t$$

then

$$\begin{aligned}\mathbb{U}_t &= -\frac{1}{\gamma}\left(1+\gamma\rho\eta_t\right)e^{-\gamma\left(x_t - \mu_t\left(B_t - \frac{\vartheta\varkappa}{(1-\vartheta)}\varphi_t\right)\right)}\Sigma_t \\ &= -\frac{1}{\gamma}\left(1+\gamma\rho\eta_t\right)e^{-\gamma x_t}S_t\end{aligned}$$

Use (69) to rewrite (64)

$$S_t = \left(e^{-\frac{\gamma}{\vartheta}W_t}\vartheta S_{t-1} + 1 - \vartheta\right)e^{\gamma W_t}e^{\frac{1}{2}\gamma^2\mu_t^2\eta_t^2\sigma_t^2} \tag{70}$$

This is the recursive definition of consumption inequality used in the definition of social welfare in the paper.

F.3 Asset Distributions

The individual budget constraint can be written as

$$\begin{aligned}A_{t+1}^s(i) &= \frac{R_t}{\vartheta}\left(\upsilon_t A_t^s(i) + Z_t + \upsilon_t\eta_t\tilde{\xi}_t^s(i)\right) \\ \left(A_{t+1}^s(i)\right)^2 &= \left(\frac{R_t}{\vartheta}\right)^2\left(\begin{array}{c}\upsilon_t^2\left(A_t^s(i)\right)^2 + 2Z_t\upsilon_tA_t^s(i) + Z_t^2 \\ +2Z_t\upsilon_t\eta_t\tilde{\xi}_t^s(i) + \upsilon_t^2\eta_t^2\left(\tilde{\xi}_t^s(i)\right)^2 + 2\upsilon_t^2\eta_t\tilde{\xi}_t^s(i)A_t^s(i)\end{array}\right)\end{aligned}$$

where

$$\begin{aligned}A_t^s(i) &= \frac{\tilde{P}_{t-1}^M}{\tilde{q}_{t-1}}a_t^{L,s}(i) + a_t^{S,s}(i) = \left(1+\varrho\tilde{P}_t^M\right)a_t^{L,s}(i) + a_t^{S,s}(i) \\ \upsilon_t &= \left(1-\left(1+\rho\gamma\eta_t\right)\mu_t\right) \\ Z_t &= \eta_t\left(\rho\log(\upsilon_t) + \bar{\xi} - \rho\gamma\chi G_t\right) + d_t - T_t - \left(1+\rho\gamma\eta_t\right)\left(\mathcal{C}_t - \chi G_t\right) \\ \tilde{\xi}_t^s(i) &= \xi_t^s(i) - \bar{\xi}\end{aligned}$$

Therefore, the mean and mean square of assets of a cohort born at time s can be described at time t by

$$\mathbb{E}_A(t,s) = \frac{R_{t-1}}{\vartheta}\left(\upsilon_{t-1}\mathbb{E}_A(t-1,s) + Z_{t-1}\right)$$
$$\mathbb{E}_{A^2}(t,s) = \left(\frac{R_{t-1}}{\vartheta}\right)^2\left(\upsilon_{t-1}^2\mathbb{E}_{A^2}(t-1,s) + 2Z_{t-1}\upsilon_{t-1}\mathbb{E}_A(t-1,s) + Z_{t-1}^2 + \upsilon_{t-1}^2\eta_{t-1}^2\sigma_{t-1}^2\right)$$

Variance

$$\begin{aligned}
\mathbb{S}_A^2(t,s) &= \mathbb{E}_{A^2}(t,s) - \left(\mathbb{E}_A(t,s)\right)^2 \\
&= \left(\frac{R_{t-1}}{\vartheta}\right)^2\left(\upsilon_{t-1}^2\mathbb{E}_{A^2}(t-1,s) + 2Z_{t-1}\upsilon_{t-1}\mathbb{E}_A(t-1,s) + Z_{t-1}^2 + \upsilon_{t-1}^2\eta_{t-1}^2\sigma_{t-1}^2\right) \\
&- \left(\frac{R_{t-1}}{\vartheta}\left(\upsilon_{t-1}\mathbb{E}_A(t-1,s) + Z_{t-1}\right)\right)^2 \\
&= \left(\frac{R_{t-1}}{\vartheta}\right)^2\left(\upsilon_{t-1}^2\mathbb{E}_{A^2}(t-1,s) + \upsilon_{t-1}^2\eta_{t-1}^2\sigma_{t-1}^2\right) - \frac{R_{t-1}^2}{\vartheta^2}\left(\upsilon_{t-1}^2\mathbb{E}_A(t-1,s)^2\right) \\
&= \upsilon_{t-1}^2\left(\frac{R_{t-1}}{\vartheta}\right)^2\left(\mathbb{S}_A^2(t-1,s) + \eta_{t-1}^2\sigma_{t-1}^2\right)
\end{aligned}$$

In steady state,

$$\upsilon\frac{R}{\vartheta} = 1$$

and if s is an age of a cohort, then

$$\mathbb{E}_A(s) = \mathbb{E}_A(s-1) + \frac{RZ}{\vartheta} = s\frac{ZR}{\vartheta} \tag{71}$$
$$\not<_A(s)^2 = \not<_A(s-1)^2 + \eta^2\sigma^2 = s\eta^2\sigma^2 \tag{72}$$

F.4 Initial Inequality

For date 0 we have

$$\Sigma_0 = (1-\vartheta)\sum_{s=-\infty}^{-1}\vartheta^{-s}e^{\gamma\mu_0\varkappa(-s)\varphi_0}I_0 + (1-\vartheta)\,e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2}$$

where

$$
\begin{aligned}
I_0 &= \int_0^1 e^{-\gamma\mu_0\left(A_0^s(i)+\eta_0\left(\xi_0^s(i)-\bar{\xi}\right)\right)}di \\
&= \int_0^1 e^{-\gamma\mu_0\eta_0\left(\xi_0^s(j)-\bar{\xi}\right)}dj \int_0^1 e^{-\gamma\mu_0\left(A_0^s(i)\right)}di \\
&= e^{+\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2}\int_0^1 e^{-\gamma\mu_0 A_0^s(i)}di \\
&= e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2}e^{s\frac{ZR}{\vartheta}\gamma\mu_0}\int_0^1 e^{-\gamma\mu_0\left(A_0^s(i)+s\frac{ZR}{\vartheta}\right)}di \\
&= e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2}e^{s\frac{ZR}{\vartheta}\gamma\mu_0}\int_0^1 e^{-\not\kappa_A(s)\gamma\mu_0\left[\frac{\left(A_0^s(i)+s\frac{ZR}{\vartheta}\right)}{\not\kappa_A(s)}\right]}di \\
&= e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2}e^{s\frac{ZR}{\vartheta}\gamma\mu_0}e^{\frac{1}{2}(\not\kappa_A(s))^2\gamma^2\mu_0^2} \\
&= e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2}e^{\left[-\frac{1}{2}\eta^2\sigma^2\gamma^2\mu_0^2+\gamma\mu_0\frac{ZR}{\vartheta}\right]s}
\end{aligned}
$$

where we used

$$
\begin{aligned}
\mathbb{E}_A(s) &= -s\frac{ZR}{\vartheta} \\
\not\kappa_A(s)^2 &= -s\eta^2\sigma^2
\end{aligned}
$$

and s is the cohort number, $s<0$.

Therefore,

$$
\begin{aligned}
\Sigma_0 &= (1-\vartheta)\sum_{s=-\infty}^{-1}\vartheta^{-s}e^{\gamma\mu_0\varkappa(-s)\varphi_0}I_0 + (1-\vartheta)\,e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2} \\
&= (1-\vartheta)\sum_{s=-\infty}^{-1}\vartheta^{-s}e^{\gamma\mu_0\varkappa(-s)\varphi_0}e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2}e^{\left[-\frac{1}{2}\eta^2\sigma^2\gamma^2\mu_0^2+\gamma\mu_0\frac{ZR}{\vartheta}\right]s} + (1-\vartheta)\,e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2} \\
&= e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2}(1-\vartheta)\sum_{s=-\infty}^{-1}\vartheta^{-s}e^{\left[-\frac{1}{2}\eta^2\sigma^2\gamma^2\mu_0^2+\gamma\mu_0\frac{ZR}{\vartheta}-\gamma\mu_0\varkappa\varphi_0\right]s} + (1-\vartheta)\,e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2} \\
&= e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2}(1-\vartheta)\,\vartheta e^{\left[\frac{1}{2}\eta^2\sigma^2\gamma^2\mu_0^2-\gamma\mu_0\frac{ZR}{\vartheta}+\gamma\mu_0\varkappa\varphi_0\right]} \\
&\times\sum_{s=0}^{\infty}\left(\vartheta e^{\left[\frac{1}{2}\eta^2\sigma^2\gamma^2\mu_0^2-\gamma\mu_0\frac{ZR}{\vartheta}+\gamma\mu_0\varkappa\varphi_0\right]}\right)^s + (1-\vartheta)\,e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2} \\
&= e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2}(1-\vartheta)\frac{\vartheta e^{\left[\frac{1}{2}\eta^2\sigma^2\gamma^2\mu_0^2-\gamma\mu_0\frac{ZR}{\vartheta}+\gamma\mu_0\varkappa\varphi_0\right]}}{1-\vartheta e^{\left[\frac{1}{2}\eta^2\sigma^2\gamma^2\mu_0^2-\gamma\mu_0\frac{ZR}{\vartheta}+\gamma\mu_0\varkappa\varphi_0\right]}} + (1-\vartheta)\,e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2} \\
&= (1-\vartheta)\frac{e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2}}{1-\vartheta e^{\left[\frac{1}{2}\eta^2\sigma^2\gamma^2\mu_0^2-\gamma\mu_0\frac{ZR}{\vartheta}+\gamma\mu_0\varkappa\varphi_0\right]}}
\end{aligned}
$$

Finally

$$
\begin{aligned}
\Sigma_0 &= (1-\vartheta) \sum_{s=-\infty}^{-1} \vartheta^{-s} e^{\gamma\mu_0\varkappa(-s)\varphi_0} I_0 + (1-\vartheta)\, e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2} \\
&= (1-\vartheta)\, \frac{e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2}}{1-\vartheta e^{\left[\frac{1}{2}\eta^2\sigma^2\gamma^2\mu_0^2 - \gamma\mu_0\frac{ZR}{\vartheta} + \gamma\mu_0\varkappa\varphi_0\right]}}.
\end{aligned}
$$

In the monetary case $Z = \varkappa = 0$

$$
\Sigma_0 = \frac{(1-\vartheta)\, e^{\frac{1}{2}\gamma^2\mu_0^2\eta_0^2\sigma_0^2}}{1-\vartheta e^{\frac{1}{2}\left(\frac{\mu_0}{\mu}\right)^2 \eta^2\sigma^2\gamma^2\mu^2}}
$$

this is the same formula as in Acharya et. al. (2022).

G Optimal Policy Under Commitment

The Lagrangian is

$$
\begin{aligned}
L = &\sum_{t=0}^{\infty}\tilde{\beta}^{t}\left(-\frac{1}{\gamma}\left(1+\gamma\rho\eta_{t}\right)^{\psi}\exp\left(-\psi\gamma x_{t}\right)S_{t}^{\xi}\right)\\
&+\sum_{t=0}^{\infty}\tilde{\beta}^{t}M_{1,t}\left(\begin{array}{c}-\frac{1}{\gamma}\log\left(\beta R_{t}\right)+x_{t+1}+\frac{(1-\vartheta)}{\vartheta}\mu_{t+1}\frac{\left(1+\varrho P_{t+1}^{M}\right)b_{t+1}^{L}}{\left(1+\pi_{t+1}\right)}-\varkappa\mu_{t+1}\varphi_{t+1}\\ -\frac{\gamma}{2}\mu_{t+1}^{2}\left(1-\tau_{t+1}\right)^{2}\omega_{t+1}-x_{t}\end{array}\right)\\
&+\sum_{t=0}^{\infty}\tilde{\beta}^{t}M_{2,t}\left(\frac{1-\varepsilon_{t}+(1-s)\,\varepsilon_{t}\frac{w_{t}}{z_{t}}}{\Phi}Y_{t}\beta^{-1}-\pi_{t}\left(1+\pi_{t}\right)Y_{t}\beta^{-1}+\pi_{t+1}\left(1+\pi_{t+1}\right)Y_{t+1}\right)\\
&+\sum_{t=0}^{\infty}\tilde{\beta}^{t}M_{3,t}\left(\frac{\vartheta}{\mu_{t+1}}+\left(1+\rho\gamma\eta_{t}\right)R_{t}-\frac{R_{t}}{\mu_{t}}\right)\\
&+\sum_{t=0}^{\infty}\tilde{\beta}^{t}M_{4,t}\left(-\log S_{t}+\frac{1}{2}\gamma^{2}\mu_{t}^{2}\left(1-\tau_{t}\right)^{2}\omega_{t}+\gamma W_{t}+\log\left(e^{-\frac{\gamma}{\vartheta}W_{t}}\vartheta S_{t-1}+1-\vartheta\right)\right)\\
&+\sum_{t=0}^{\infty}\tilde{\beta}^{t}M_{5,t}\left(\left(\frac{\left(1+\varrho P_{t}^{M}\right)b_{t}^{L}}{\left(1+\pi_{t}\right)}+G_{t}-\tau_{t}\frac{w_{t}}{z_{t}}Y_{t}-T_{t}^{p}\right)R_{t}-\frac{\left(1+\varrho P_{t+1}^{M}\right)b_{t+1}^{L}}{\left(1+\pi_{t+1}\right)}\right)\\
&+\sum_{t=0}^{\infty}\tilde{\beta}^{t}M_{6,t}\left(\rho\log\left(\eta_{t}\right)+\bar{\xi}-\frac{\varkappa\vartheta}{1-\vartheta}-\rho\gamma x_{t}-\frac{Y_{t}}{z_{t}}\right)+\sum_{t=0}^{\infty}\tilde{\beta}^{t}M_{7,t}\left(\left(1-\tau_{t}\right)w_{t}-\eta_{t}\right)\\
&+\sum_{t=0}^{\infty}\tilde{\beta}^{t}M_{8,t}\left(\frac{\left(1+\varrho P_{t+1}^{M}\right)}{\left(1+\pi_{t+1}\right)}-P_{t}^{M}R_{t}\right)+\sum_{t=0}^{\infty}\tilde{\beta}^{t}M_{9,t}\left(x_{t}+\left(1-\chi\right)G_{t}-\left(1-\frac{\Phi}{2}\pi_{t}^{2}\right)Y_{t}\right)\\
&+\sum_{t=0}^{\infty}\tilde{\beta}^{t}M_{10,t}\left(W_{t}-\mu_{t}\left(\frac{\left(1+\varrho P_{t}^{M}\right)b_{t}^{L}}{\left(1+\pi_{t}\right)}-\frac{\vartheta}{\left(1-\vartheta\right)}\varkappa\varphi_{t}\right)\right)\\
&+\sum_{t=0}^{\infty}\tilde{\beta}^{t}M_{11,t}\left(R_{t}\eta_{t}+\vartheta\varphi_{t+1}-R_{t}\varphi_{t}\right)+\sum_{t=0}^{\infty}\tilde{\beta}^{t}M_{12,t}\left(w^{2}\sigma^{2}\exp\left(2\phi\left(Y_{t}-Y\right)\right)-\omega_{t}\right)
\end{aligned}
$$

and the FOCs are

$$
1:\frac{\partial L}{\partial\varphi_{t}}=-\tilde{\beta}^{-1}M_{1,t-1}\varkappa\mu_{t}+M_{10,t}\mu_{t}\frac{\vartheta\varkappa}{\left(1-\vartheta\right)}-M_{11,t}R_{t}+\tilde{\beta}^{-1}M_{11,t-1}\vartheta
$$

$$
\begin{aligned}
2:\frac{\partial L}{\partial\mu_{t}}=&\,\tilde{\beta}^{-1}M_{1,t-1}\left(\frac{(1-\vartheta)}{\vartheta}\frac{\left(1+\varrho P_{t}^{M}\right)b_{t}^{L}}{\left(1+\pi_{t}\right)}-\varkappa\varphi_{t}-\gamma\mu_{t}\left(1-\tau_{t}\right)^{2}\omega_{t}\right)\\
&+M_{3,t}\frac{R_{t}}{\mu_{t}^{2}}-\tilde{\beta}^{-1}M_{3,t-1}\frac{\vartheta}{\mu_{t}^{2}}+M_{4,t}\gamma^{2}\mu_{t}\left(1-\tau_{t}\right)^{2}\omega_{t}\\
&-M_{10,t}\left(\frac{\left(1+\varrho P_{t}^{M}\right)b_{t}^{L}}{\left(1+\pi_{t}\right)}-\frac{\vartheta\varkappa}{\left(1-\vartheta\right)}\varphi_{t}\right)
\end{aligned}
$$

$$
3:\frac{\partial L}{\partial w_{t}}=M_{2,t}\frac{(1-s)}{\Phi}\frac{\varepsilon_{t}}{z_{t}}Y_{t}\beta^{-1}-M_{5,t}\frac{\tau_{t}}{z_{t}}Y_{t}R_{t}+M_{7,t}\left(1-\tau_{t}\right)
$$

$$4:\frac{\partial L}{\partial \eta_t}=-\rho\psi S_t^{\xi}\left(1+\gamma\rho\eta_t\right)^{\psi-1}\exp\left(-\psi\gamma x_t\right)$$
$$+M_{3,t}\rho\gamma R_t+M_{6,t}\frac{\rho}{\eta_t}-M_{7,t}+M_{11,t}R_t$$

$$5:\frac{\partial L}{\partial R_t}=-M_{1,t}\frac{1}{\gamma R_t}+M_{3,t}\left(1+\rho\gamma\eta_t-\frac{1}{\mu_t}\right)$$
$$+M_{5,t}\left(\frac{\left(1+\varrho P_t^M\right)b_t^L}{\left(1+\pi_t\right)}+G_t-\tau_t\frac{w_t}{z_t}Y_t-T_t^p\right)$$
$$-M_{8,t}P_t^M+M_{11,t}\left(\eta_t-\varphi_t\right)$$

$$6:\frac{\partial L}{\partial b_{t+1}^L}=M_{1,t}\frac{(1-\vartheta)}{\vartheta}\mu_{t+1}\frac{\left(1+\varrho P_{t+1}^M\right)}{\left(1+\pi_{t+1}\right)}-M_{5,t}\frac{\left(1+\varrho P_{t+1}^M\right)}{\left(1+\pi_{t+1}\right)}$$
$$+\tilde{\beta}M_{5,t+1}R_{t+1}\frac{\left(1+\varrho P_{t+1}^M\right)}{\left(1+\pi_{t+1}\right)}-\tilde{\beta}M_{10,t+1}\mu_{t+1}\frac{\left(1+\varrho P_{t+1}^M\right)}{\left(1+\pi_{t+1}\right)}$$

$$7:\frac{\partial L}{\partial P_t^M}=\left(\tilde{\beta}^{-1}M_{1,t-1}\frac{(1-\vartheta)}{\vartheta}\mu_t+M_{5,t}R_t-\tilde{\beta}^{-1}M_{5,t-1}-M_{10,t}\mu_t\right)\frac{\varrho b_t^L}{\left(1+\pi_t\right)}$$
$$-M_{8,t}R_t+\tilde{\beta}^{-1}M_{8,t-1}\frac{\varrho}{\left(1+\pi_t\right)}$$

$$8:\frac{\partial L}{\partial S_t}=-\frac{1}{\gamma}\left(1+\gamma\rho\eta_t\right)^{\psi}\exp\left(-\psi\gamma x_t\right)\xi S_t^{\xi-1}$$
$$-M_{4,t}\frac{1}{S_t}+\tilde{\beta}M_{4,t+1}\frac{e^{-\frac{\gamma}{\vartheta}W_{t+1}}\vartheta}{\left(e^{-\frac{\gamma}{\vartheta}W_{t+1}}\vartheta S_t+1-\vartheta\right)}$$

$$9:\frac{\partial L}{\partial W_t}=M_{4,t}\gamma\left(1-\frac{e^{-\frac{\gamma}{\vartheta}W_t}S_{t-1}}{\left(e^{-\frac{\gamma}{\vartheta}W_t}\vartheta S_{t-1}+1-\vartheta\right)}\right)+M_{10,t}$$

$$10:\frac{\partial L}{\partial \pi_t}=-\tilde{\beta}^{-1}M_{1,t-1}\frac{(1-\vartheta)}{\vartheta}\mu_t\frac{\left(1+\varrho P_t^M\right)b_t^L}{\left(1+\pi_t\right)^2}-M_{2,t}\left(1+2\pi_t\right)Y_t\beta^{-1}+\tilde{\beta}^{-1}M_{2,t-1}\left(1+2\pi_t\right)Y_t$$
$$-M_{5,t}R_t\frac{\left(1+\varrho P_t^M\right)b_t^L}{\left(1+\pi_t\right)^2}+\tilde{\beta}^{-1}M_{5,t-1}\frac{\left(1+\varrho P_t^M\right)b_t^L}{\left(1+\pi_t\right)^2}$$
$$-\tilde{\beta}^{-1}M_{8,t-1}\frac{\left(1+\varrho P_t^M\right)}{\left(1+\pi_t\right)^2}+M_{9,t}\Phi\pi_tY_t+M_{10,t}\mu_t\frac{\left(1+\varrho P_t^M\right)b_t^L}{\left(1+\pi_t\right)^2}$$

$$11:\frac{\partial L}{\partial x_t}=\psi\left(1+\gamma\rho\eta_t\right)^{\psi}\exp\left(-\psi\gamma x_t\right)S_t^{\xi}-M_{1,t}+\tilde{\beta}^{-1}M_{1,t-1}-M_{6,t}\rho\gamma+M_{9,t}$$

$$12:\frac{\partial L}{\partial Y_t}=+M_{2,t}\left(\frac{1-\varepsilon_t+(1-s)\,\varepsilon_t\frac{w_t}{z_t}}{\Phi}-\pi_t\,(1+\pi_t)\right)\beta^{-1}$$
$$+\tilde{\beta}^{-1}M_{2,t-1}\pi_t\,(1+\pi_t)-M_{5,t}\tau_t\frac{w_t}{z_t}R_t-M_{6,t}\frac{1}{z_t}$$
$$-M_{9,t}\left(1-\frac{\Phi}{2}\pi_t^2\right)+2\phi M_{12,t}w^2\sigma^2\exp\left(2\phi\left(Y_t-Y\right)\right)$$

$$13a:\frac{\partial L}{\partial \tau_t}=\tilde{\beta}^{-1}M_{1,t-1}\gamma\mu_t^2\,(1-\tau_t)\,\omega_t-M_{4,t}\gamma^2\mu_t^2\,(1-\tau_t)\,\omega_t-M_{5,t}\frac{w_t}{z_t}Y_tR_t-M_{7,t}w_t$$
$$13b:\frac{\partial L}{\partial T_t^p}=-M_{5,t}R_t$$

here equation 13a is for the case of labor income taxation, and equation 13b applies when there are lump sum taxes.

G.1 Steady State

The FOCs in the steady state can be written as,

$$1:\frac{\partial L}{\partial \varphi_t}=-\tilde{\beta}^{-1}M_1\varkappa\mu+M_{10}\mu\frac{\vartheta\varkappa}{(1-\vartheta)}+M_{11}\left(\vartheta\tilde{\beta}^{-1}-R\right)$$

$$2:\frac{\partial L}{\partial \mu_t}=\tilde{\beta}^{-1}M_1\left(\frac{(1-\vartheta)}{\vartheta}\frac{\left(1+\varrho P^M\right)b^L}{(1+\pi_t)}-\varkappa\varphi-\gamma\mu\,(1-\tau)^2\,\omega\right)$$
$$+M_3\frac{R}{\mu^2}-\tilde{\beta}^{-1}M_3\frac{\vartheta}{\mu^2}+M_4\gamma^2\mu\,(1-\tau)^2\,\omega$$
$$-M_{10}\left(\frac{\left(1+\varrho P^M\right)b^L}{(1+\pi)}-\frac{\vartheta\varkappa}{(1-\vartheta)}\varphi\right)$$

$$3:\frac{\partial L}{\partial w_t}=M_2\frac{(1-s)}{\Phi}\varepsilon Y\beta^{-1}-M_5\tau YR+M_7\,(1-\tau)$$

$$4:\frac{\partial L}{\partial \eta_t}=-\rho\psi S^{\xi}\,(1+\gamma\rho\eta)^{\psi-1}\exp\left(-\psi\gamma x\right)$$
$$+M_3\rho\gamma R+M_6\frac{\rho}{\eta}-M_7+M_{11}R$$

$$5:\frac{\partial L}{\partial R_t}=-M_1\frac{1}{\gamma R}+M_3\left(1+\rho\gamma\eta-\frac{1}{\mu}\right)$$
$$+M_5\left(\frac{\left(1+\varrho P^M\right)b^L}{(1+\pi)}+G-\tau wY-T^p\right)$$
$$-M_8P^M+M_{11}\left(\eta-\varphi\right)$$

$$6:\frac{\partial L}{\partial b_{t+1}^{L}}=M_1\frac{(1-\vartheta)}{\vartheta}\mu\frac{\left(1+\varrho P^{M}\right)}{(1+\pi)}-M_5\frac{\left(1+\varrho P^{M}\right)}{(1+\pi)}$$
$$+\tilde{\beta}M_5R\frac{\left(1+\varrho P^{M}\right)}{(1+\pi)}-\tilde{\beta}M_{10}\mu\frac{\left(1+\varrho P^{M}\right)}{(1+\pi)}$$

$$7:\frac{\partial L}{\partial P_t^{M}}=\left(\tilde{\beta}^{-1}M_1\frac{(1-\vartheta)}{\vartheta}\mu+M_5R-\tilde{\beta}^{-1}M_5-M_{10}\mu\right)\frac{\varrho b^{L}}{(1+\pi)}$$
$$-M_8R+\tilde{\beta}^{-1}M_8\frac{\varrho}{(1+\pi)}$$

$$8:\frac{\partial L}{\partial S_t}=-\frac{1}{\gamma}\left(1+\gamma\rho\eta\right)^{\psi}\exp\left(-\psi\gamma x\right)\xi S^{\xi-1}$$
$$-M_4\frac{1}{S}+\tilde{\beta}M_4\frac{e^{-\frac{\gamma}{\vartheta}W}\vartheta}{\left(e^{-\frac{\gamma}{\vartheta}W}\vartheta S+1-\vartheta\right)}$$

$$9:\frac{\partial L}{\partial W_t}=M_4\gamma\left(1-\frac{e^{-\frac{\gamma}{\vartheta}W}S}{\left(e^{-\frac{\gamma}{\vartheta}W}\vartheta S+1-\vartheta\right)}\right)+M_{10}$$

$$10:\frac{\partial L}{\partial \pi_t}=-\tilde{\beta}^{-1}M_1\frac{(1-\vartheta)}{\vartheta}\mu\frac{\left(1+\varrho P^{M}\right)b^{L}}{(1+\pi)^2}-M_2\left(1+2\pi\right)Y\beta^{-1}+\tilde{\beta}^{-1}M_2\left(1+2\pi\right)Y$$
$$-M_5R\frac{\left(1+\varrho P^{M}\right)b^{L}}{(1+\pi)^2}+\tilde{\beta}^{-1}M_5\frac{\left(1+\varrho P^{M}\right)b^{L}}{(1+\pi)^2}$$
$$-\tilde{\beta}^{-1}M_8\frac{\left(1+\varrho P^{M}\right)}{(1+\pi)^2}+M_9\Phi\pi Y+M_{10}\mu\frac{\left(1+\varrho P^{M}\right)b^{L}}{(1+\pi)^2}$$

$$11:\frac{\partial L}{\partial x_t}=\psi\left(1+\gamma\rho\eta\right)^{\psi}\exp\left(-\psi\gamma x\right)S^{\xi}-M_1+\tilde{\beta}^{-1}M_1-M_6\rho\gamma+M_9$$

$$12:\frac{\partial L}{\partial Y_t}=+M_2\left(\frac{1-\varepsilon+(1-s)\,\varepsilon w}{\Phi}-\pi\left(1+\pi\right)\right)\beta^{-1}$$
$$+\tilde{\beta}^{-1}M_2\pi\left(1+\pi\right)-M_5\tau wR-M_6$$
$$-M_9\left(1-\frac{\Phi}{2}\pi^2\right)+2\phi M_{12}w^2\sigma^2$$

$$13a:\frac{\partial L}{\partial \tau_t}=\tilde{\beta}^{-1}M_1\gamma\mu^2\left(1-\tau\right)\omega-M_4\gamma^2\mu^2\left(1-\tau\right)\omega-M_5wYR-M_7w$$

$$13b:\frac{\partial L}{\partial T_t^{p}}=-M_5R$$

Additionally, the system of constraints can be written as follows.

$$
\begin{aligned}
14 &: 0 = -\frac{1}{\gamma}\log\left(\beta R\right) + \frac{(1-\vartheta)}{\vartheta}\mu\frac{\left(1+\varrho P^M\right)b^L}{(1+\pi)} - \varkappa\mu\varphi - \frac{\gamma}{2}\mu^2\eta^2\sigma^2 \\
15 &: 0 = \frac{1-\varepsilon_t + (1-s)\,\varepsilon w}{\Phi}Y\beta^{-1} - \pi\left(1+\pi\right)Y\beta^{-1} + \pi\left(1+\pi\right)Y \\
16 &: 0 = \frac{\vartheta\mu}{\left(\mu\left(1+\rho\gamma\eta\right)-1\right)R} + \mu \\
17 &: 0 = -\log S + \frac{1}{2}\gamma^2\mu^2\eta^2\sigma^2 + \gamma W + \log\left(e^{-\frac{\gamma}{\vartheta}W}\vartheta S + 1 - \vartheta\right) \\
18 &: 0 = \left(\frac{\left(1+\varrho P^M\right)b^L}{(1+\pi)} + G - \tau w Y - T^p\right)R - \frac{\left(1+\varrho P^M\right)b^L}{(1+\pi)} \\
19 &: 0 = \rho\log\left(\eta\right) + \bar{\xi} - \frac{\varkappa\vartheta}{1-\vartheta} - \rho\gamma x - Y \\
20 &: 0 = \left(1-\tau\right)w - \eta \\
21 &: 0 = \frac{\left(1+\varrho P^M\right)}{(1+\pi)} - P^M R \\
22 &: 0 = x + \left(1-\chi\right)G - \left(1 - \frac{\Phi}{2}\pi^2\right)Y \\
23 &: 0 = W - \mu\left(\frac{\left(1+\varrho P^M\right)b^L}{(1+\pi)} - \frac{\vartheta\varkappa}{(1-\vartheta)}\varphi\right) \\
24 &: 0 = R\eta + \vartheta\varphi - R\varphi
\end{aligned}
$$

H Proof of Proposition 3

When we have lump-sum taxes T_t^p is a policy instrument, the budget constraint (29) does not bind in the steady state, and $M_5 = 0$ as follows from the FOC wrt T_t^p. From the FOC wrt b_{t+1}^L it follows that,

$$M_{10} = M_1\frac{(1-\vartheta)}{\vartheta\tilde{\beta}} \tag{73}$$

then using this in the FOC wrt P_t^M we get $M_8 = 0$ meaning that equation (33), the definition of bond prices, also is not a binding constraint in steady state.

Use (73) to substitute M_{10} into the FOC wrt to φ, to yield $M_{11} = 0$. Then, in case of RANK such that $\sigma = \omega = 0$ we use (73) in the FOC wrt μ to yield $M_3 = 0$. While, the FOC wrt R yields $M_1 = M_{10} = 0$.

The FOC wrt S implies that $M_4 \neq 0$ as the derivative of utility is never zero. Finally, the

FOC wrt W can be written as,

$$\frac{\partial L}{\partial W_t} = M_4 \gamma \frac{(1-\vartheta)\left(1 - e^{-\frac{\gamma}{\vartheta}W} S\right)}{\left(e^{-\frac{\gamma}{\vartheta}W} \vartheta S + 1 - \vartheta\right)} = 0$$

from where

$$S = e^{\frac{\gamma}{\vartheta}W}$$

Substituting this into the evolution of S equation yields

$$0 = -\frac{\gamma}{\vartheta}W + \gamma W$$

so that $W = 0$, and $S = 1$.

The Euler equation then implies

$$R = \frac{1}{\beta}.$$

All other results in Proposition 3 follow trivially.

It is essential that $\sigma = 0$, the result will break down otherwise. We see that the level of debt in the steady state will be determined by the rate at which labor force participation declines with age.

I Proof of Proposition 4

We begin by assuming $\tilde{\beta} = \beta = \frac{1}{R}$, $\sigma = 0$ so that $\omega = 0$. When then show that a Ramsey steady-state with these features can only exist if there is no inequality, no government debt and no retirement.

Suppose $R = \frac{1}{\beta}$ then Euler equation (26) yields $W = 0$ and from inequality recursion (38) it follows that $S = 1$.The FOC wrt b_{t+1}^L yields $M_{10} = M_1 \frac{(1-\vartheta)}{\vartheta\tilde{\beta}}$. Use this in the FOC wrt P_t^M to yield $M_8 = 0$.Substitute $M_{10} = M_1 \frac{(1-\vartheta)}{\vartheta\tilde{\beta}}$ into the FOC wrt to φ, to yield $M_{11} = 0$.Then, the FOC wrt μ yields $M_3 = 0$, once we take into account the relationship between M_{10} and M_1.

The FOC wrt S implies

$$M_4 = \frac{(1+\gamma\rho\eta)}{\gamma(\beta\vartheta - 1)} \exp(-\gamma x)$$

so that $M_4 \neq 0$.

The FOC wrt W, taking into account that $S = 1$, yields $M_{10} = 0$, so that, from above, $M_1 = 0$.

Now, consider the system of FOCs wrt $\tau_t, w_t, Y_t, \eta_t, x_t$. This is a linear system wrt the five Lagrange multipliers: M_7, M_2, M_6, M_9, M_5. We can show that this system has a unique solution and all these multipliers are non-zero, in general.

Specifically, FOC wrt τ_t yields $M_7 = -M_5 Y R$, then FOC wrt w_t yields $M_2 = M_5 \frac{\Phi}{(1-s)\varepsilon}$. The remaining three equations are more complex, but we can substitute out M_6 and M_9 to arrive at,

$$M_5 = \frac{\rho\left(\eta - \left(1 - \frac{\Phi}{2}\pi^2\right)\right)\exp\left(-\gamma x\right) S}{R\left(\left(1 - \frac{\Phi}{2}\pi^2\right)\rho\gamma\eta Y + \rho\left(\frac{1-\varepsilon+(1-s)\varepsilon w}{(1-s)\varepsilon} - \tau w\right) + \eta Y\right)}$$

from which it is clear that, generally speaking, $M_5 \neq 0$, and therefore $M_9 \neq 0$ and $M_6 \neq 0$.

Then, the FOC wrt π_t yields $M_9 \Phi \pi Y = 0$, from which it must be the case that $\pi = 0$ in the steady state.

The FOC wrt R_t yields

$$\begin{aligned}\frac{\partial L}{\partial R_t} &= M_5\left(\frac{\left(1+\varrho P^M\right) b^L}{(1+\pi)} + G - \tau w Y - T^p\right)\\ &= M_5 \frac{\left(1+\varrho P^M\right)}{R\left(1+\pi\right)} b^L\end{aligned}$$

which implies $b^L = 0$.

Finally, because $W = b^L = 0$, then the definition of W,

$$W = \mu\left(\frac{\left(1+\varrho P^M\right)}{(1+\pi)} b^L - \frac{\vartheta\varphi}{(1-\vartheta)}\varkappa\right),$$

means that this can only hold if $\varkappa = 0$, as $\varphi = \frac{R\eta}{(R-\vartheta)} \neq 0$.