The inflation risk premium on government debt in an overlapping generations model

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Abstract

An overlapping generations model with long run inflation risk and a cash-in-advance constraint is used to derive a second-order accurate closed-form solution for the inflation risk premium on long-term government debt. The model predicts that the inflation risk premium depends crucially on the relative importance of nominal bonds and capital as sources of retirement consumption. In a calibrated model the predicted risk premium is non-trivial under plausible levels of risk aversion. The cash-in-advance constraint is crucial for this result.

Keywords: government debt; inflation risk premium; overlapping generations.

1 Introduction

The inflation risk premium – the compensation demanded by risk-averse nominal bondholders for bearing inflation risk – is of clear practical importance. For instance, the question of whether it is optimal to issue indexed government debt will depend in part on the cost of financing debt repayments. Other things being equal, a positive inflation risk premium implies that government could borrow more cheaply by issuing debt that is indexed to the price level. This strategy would enable the government to finance more spending for any given path of taxes, or to keep spending unchanged in real terms while permanently lowering taxes. Neither of these effects is likely to be trivial. Moreover, as Bernanke (2004) notes, estimates of the inflation risk premium enable policymakers to make inferences about inflation expectations using break-even inflation rates – the difference between nominal and indexed yields on bonds of the same maturity. Hence, the inflation risk premium matters.

In a recent survey, Bekaert and Wang (2010) note that most empirical estimates of inflation risk premia are robustly positive, with some recent estimates being as large as 100 basis points. To better understand the economic factors that drive inflation risk premia, several recent papers have computed inflation risk-premia in non-linear New Keynesian models (Andreasen, 2012; De Paoli et al., 2010; Hördahl et al., 2008; Ravenna and Seppälä, 2007). Because nominal prices are sticky in these models, monetary policy has real effects. As a result, the inflation risk premium – the covariance between the stochastic discount factor and inflation – depends crucially on the shocks that hit the economy and the way that monetary policy responds to these shocks. For example, using a standard New Keynesian model, De Paoli et al. (2010) find that the sign of the inflation risk premium depends crucially on whether the economy is dominated by productivity or monetary policy shocks: it is positive in a world dominated by productivity shocks but negative when monetary policy shocks dominate. More recently, Andreasen (2012) estimates a medium-scale New Keynesian model of the UK economy that allows for time-varying bond risk premia. He concludes that there was a substantial fall in nominal term premia in the 1990s caused by a fall in inflation risk premia that was driven by preference, investment, and fixed cost shocks, and a more aggressive response to inflation by the Bank of England.

This theoretical literature has improved our understanding of the factors that drive inflation risk premia, but the available studies are confined to short and medium-term maturities.[1] This paper aims to fill this gap by studying long-term inflation risk premia in a general equilibrium model with long run inflation risk. There are several good reasons to be interested in long-term maturities such as 20-30 years. First, long-term government debt plays an important role in many developed economies and accounts for a non-trivial share of total marketable government debt.[2] Second, the available empirical evidence suggests that the inflation risk premium on long-term government debt is somewhat higher than that on short-term debt (Haubrich et al., 2008; Reschreiter, 2004).[3] Third, long-term nominal interest rates matter for the transmission mechanism of monetary policy, so tracing out the impact of monetary policy on long rates is an important task for researchers and policymakers. Amongst other things, this requires an understanding of which factors matter for inflation risk premia, and why.

This paper makes two contributions that improve of understanding of long-term inflation risk premia. First, an overlapping generations model with long run inflation risk and a cash-in-advance constraint is used to derive a second-order-accurate closed-form solution for the inflation risk premium on long-term government debt.[4] The model predicts that the risk premium depends crucially on the relative importance of nominal bonds and capital as sources of retirement consumption. This finding has intuitive appeal since we would expect compensation for inflation risk to depend on the extent which households are exposed to unanticipated inflation through nominal asset holdings. In a seminal paper, Doepke and Schneider (2006) show that, in the postwar period, the US economy has been quite exposed to such fluctuations: a moderate episode of unanticipated inflation implies a substantial wealth loss for old agents, the main bondholders in the economy.[5] Likewise, the old in Canada lose out significantly during periods of unanticipated inflation, owing in part to their substantial holdings of nominal government debt (Meh and Terajima, 2011; Meh et al. 2010). The model presented here is consistent with the old being hit by unanticipated inflation because it is a standard overlapping generations model in the spirit of Champ and Freeman (1990) and Hatcher (2014) where the young save for their old age (when they are retired) by holding positive amounts of nominally-denominated government debt.

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[1] De Paoli et al. (2010) report numerical results for 1-quarter bonds only; Andreasen (2012) and Ravenna and Seppälä (2007) report results for 5-year and 10-year bonds, and Hördahl et al. (2008) report results for maturities ranging from 1 quarter up to 10 years.

[2] For example, bonds with maturities exceeding 15 years averaged around one-third of outstanding UK marketable debt over the period 2002-2012. In Canada, 30-year bonds accounted for around one-fifth of outstanding marketable debt in 2008 and an average share of around 15% over the period 2008 to 2012. Data sources: Historical Data, Debt Management Office UK; Department of Finance Canada (2011).

[3] Reschreiter (2004) does not estimate the inflation risk premium directly, but he finds that UK borrowing costs could be reduced significantly by issuing medium and long-term inflation-indexed bonds.

[4] Because the solution is second-order accurate, the inflation risk premium is constant over time.

[5] For instance, based on 1989 data, a fully unanticipated inflation of 5% that lasts 10 years produces a wealth loss relative to average net worth of 7% for the 66-75 age group and 10% for the over 75s.

The second contribution is to show that the quantitative inflation risk premium predicted by the model is non-trivial under plausible calibrations of risk aversion. In particular, the model gives an annualized inflation risk premium of around 20 basis points under the baseline calibration. This number is of the same order of magnitude as most empirical estimates of inflation risk premia and fairly robust in sensitivity analysis. By comparison, the New Keynesian literature discussed above has struggled to generate sizeable inflation risk premia under standard calibrations of risk aversion. For instance, Ravenna and Seppälä (2007), Hördahl et al. (2008) and De Paoli (2010) all find that typical calibrated New Keynesian models with external habit formation predict inflation risk premia of less than 10 basis points.[6] By contrast, Andreasen (2012) finds an average inflation risk premium on 5-year bonds of around 50 basis points in an estimated New Keynesian model of the UK economy, but this figure relies crucially on high risk aversion through Epstein-Zin preferences. Regarding empirical studies, there is a large recent literature which estimates no-arbitrage affine models of the term structure and focuses mainly on 5-year and 10-year bond maturities.[7]The bottom line from this literature is that the average inflation risk premium in the Euro Area is quite modest at around 20 basis points (e.g. Hördahl and Tristani, 2012), but both the US and UK appear to have substantial average inflation risk premia, with the most likely range being 50 to 100 basis points in the UK (Andreasen, 2012; Joyce et al., 2010) and 50 to 120 basis points in the US (e.g. Ang et al., 2008; Chernov and Mueller, 2012).

An additional finding is that the cash-in-advance constraint is crucial for producing a sizeable inflation risk premium. This is for two reasons. First, it implies that inflation is positively related to money supply growth but inversely related to retirement consumption. As a result, the stochastic discount factor and inflation are more strongly correlated than in the absence of the cash-in-advance motive. Second, the cash-in-advance constraint implies that inflation risk depends on both productivity risk and monetary volatility, whereas only monetary volatility matters when the cash-in-advance motive is absent. Consequently, inflation risk rises under cash-in-advance and pushes up the inflation risk premium.

The remainder of the paper proceeds as follows. Section 2 sets out the model. In Section 3, a closed-form analytical solution for the inflation risk premium is derived and discussed. Section 4 compares the predicted inflation risk premium with estimates in the theoretical and empirical literature. Section 5 investigates robustness. Finally, Section 6 concludes.

2. Model

The model is a version of Diamond's (1965) model where the young save for old age using government bonds and capital.[8] It contains three sectors: a household sector, a government sector, and a firm sector devoted to the production of a single output good. These sectors are discussed in this section along with the model's equilibrium conditions.

[6] Ravenna and Seppälä (2007) report negative inflation risk premia. De Paoli et al. (2010) find that the inflation risk premium can be positive or negative depending on shock composition. In Hördahl et al. (2008), the inflation risk premium is positive over a wide range of maturities but does not exceed 1 basis point.

[7] For a review, see Bekaert and Wang (2010). A brief review is also provided in Section 4.3 of the present paper.

[8] Diamond (1965) studies a real economy where consumers hold indexed government debt and capital.

2.1 Consumers

Consider a two-period overlapping generations model similar in spirit to Diamond (1965), Champ and Freeman (1990) and Hatcher (2014).[9] Each generation supplies one unit of labour in the first period, and retires in the second period. Utility is derived from consumption in both periods, and there is no bequest motive. The number of generations born in each period is normalized to 1. Consumption by the young is denoted by $c_{t,y}$. The young are subject to a lump-sum tax T_t. Their after-tax wage income can used for consumption in the same period or allocated to three assets – capital, k_{t+1}, indexed government bonds, $b_{i,t+1}$, and nominal government bonds, $b_{n,t+1}$ – in order to finance consumption in old age, $c_{t+1,o}$. In addition, the young purchase all the nominal money stock, M_t, from the current old in order to finance part of their consumption in retirement. Since there are no bequests, the old consume all their asset income. In addition, they receive lump-sum monetary transfers TR_t before they sell their money holdings to the young.[10] The nominal money stock in period t grows at rate θ_t. There are two sources of aggregate risk in the economy: a productivity shock and money supply shocks. Each period lasts N years in total.

Capital, k, is used an input in production next period, after which time its productive value is zero (i.e. there is full depreciation). Capital yields a risky return $r_{k,t+1}$. Bonds take the form of long-term government debt with maturity N years. Indexed bonds pay a riskless real return r_t, and nominal bonds a riskless nominal return R_t. These returns are endogenously determined so that the markets for indexed and nominal bonds clear.[11] Because inflation cannot be forecast with certainty, nominal bonds pay a risky real return $r_{n,t+1} = R_t / \Pi_{t+1}$, where $\Pi_{t+1} \equiv P_{t+1}/P_t$ is the rate of inflation. The real return on money balances is thus $r_{m,t+1} = 1/\Pi_{t+1}$. A positive demand for money is motivated by a cash-in-advance constraint on old age consumption. Following Hahn and Solow (1995), the old are required to purchase a fraction δ of consumption using money balances. Hence $M_{t+1} \geq \delta P_{t+1} c_{t+1,o}$, where $M_{t+1} = M_t + TR_{t+1}$ is the post-transfer level of money holdings of the old. This kind of cash-in-advance constraint has been used in several recent papers which use overlapping generations models with perfect foresight to address issues of optimal monetary policy (see e.g. Michel and Wigniolle, 2005; Gahvari, 2007). As in these papers, the cash-in-advance constraint is assumed to bind.

In real terms, the binding cash-in-advance constraint implies that

$$m_{t+1} = \delta \, c_{t+1,o}, \qquad \forall \, t \tag{1}$$

where $m_{t+1} = r_{m,t+1} m_t + TR_{real,t+1}$, $TR_{real,t} \equiv TR_t / P_t$ and $m_t = M_t / P_t$.

The budget constraints of the young born in period t are

$$c_{t,y} = w_t - T_t - k_{t+1} - b_{i,t+1} - b_{n,t+1} - m_t \tag{2}$$

$$c_{t+1,o} = r_{k,t+1} k_{t+1} + r_t b_{i,t+1} + r_{n,t+1} b_{n,t+1} + r_{m,t+1} m_t + TR_{real,t+1} \tag{3}$$

[9] In Champ and Freeman (1990) consumers hold money, capital and nominal government debt, but the inflation risk premium is zero. Hatcher (2014) considers an overlapping generations model with nominal and indexed debt and an inflation risk premium, but money is not introduced by a cash-in-advance constraint in his model.

[10] This assumption ensures that the entire money stock is passed from the current old to the current young.

[11] These returns are dated at time t because they clear the markets for bonds at the date when bonds are purchased, i.e. at end of period t.

Consumers have CRRA preferences. The young of period t solve the following problem:

$$\max_{\{k_{t+1},b_{i,t+1},b_{n,t+1}\}} U_t = \frac{c_{t,y}^{1-\gamma}-1}{1-\gamma} + \beta E_t\left[\frac{c_{t+1,o}^{1-\gamma}-1}{1-\gamma}\right] \qquad \text{s.t. (1), (2), (3)} \tag{4}$$

where β is the discount factor and γ is the coefficient of relative risk aversion.

The first-order conditions are as follows:[12]

$$1 = E_t\left[\frac{sdf_{t+1}r_{k,t+1}}{1+\delta(\Pi_{t+1}r_{k,t+1}-1)/\theta_{t+1}}\right] \qquad \text{for capital, } k \tag{5}$$

$$1 = E_t\left[\frac{sdf_{t+1}r_{n,t+1}}{1+\delta(\Pi_{t+1}r_{n,t+1}-1)/\theta_{t+1}}\right] \qquad \text{for nominal bonds, } b_n \tag{6}$$

$$1 = E_t\left[\frac{sdf_{t+1}r_t}{1+\delta(\Pi_{t+1}r_t-1)/\theta_{t+1}}\right] \qquad \text{for indexed bonds, } b_i \tag{7}$$

where $sdf_{t+1} \equiv \beta(c_{t+1,o}/c_{t,y})^{-\gamma}$ and θ_{t+1}, the money growth rate, is given in Section 2.3.2.

Notice that these Euler equations are non-standard due to the inclusion of the term multiplied by the coefficient δ, the fraction of old-age consumption that must be purchased using money. Setting $\delta = 0$ gives standard Euler equations of the form $1 = E[sdf.r_j]$ for $j = \{k, n, i\}$.

2.2 Firms

The production sector consists of a representative firm that produces output by combining capital and labour in a Cobb-Douglas production function. The share of capital in output is equal to α and the labour share is $1-\alpha$. The firm hires capital and labour in competitive markets so as to maximise current profits. Total factor productivity in period t is denoted A_t.

The real wage and the return on capital are given by:

$$w_t = y_t - r_{k,t}k_t = (1-\alpha)A_t k_t^{\alpha} \tag{8}$$

$$r_{k,t} = \alpha y_t / k_t = \alpha A_t k_t^{\alpha-1} \tag{9}$$

Productivity follows an AR(1) process in logs:

$$\ln A_t = \rho_A \ln A_{t-1} + e_t \tag{10}$$

where e_t is an IID-normal innovation with mean zero and standard deviation σ_A.

2.3 Government

The government conducts fiscal policy (taxes and transfers), sets the total supply of government debt $b = b_i + b_n$, and commits to a money supply rule. Both indexed and nominal debt are in non-negative supply, so $b_i, b_n \geq 0$. Because the analytical results that follow are

[12] A full derivation of these first-order conditions is provided in Appendix A.

general, no bond supply rule is specified at this stage. However, it should be noted that the analytical results rely on the assumption that the bond supply equation is such that the model has a unique steady-state around which the model can be approximated.

2.3.1 Fiscal policy

Real government spending is assumed, for simplicity, to be a constant fraction $0 < \phi_g < 1$ of GDP, so that $g_t = \phi_g y_t$. Given bond prices, the supply of government debt, and the path of government spending, lump-sum taxes T_t are adjusted to ensure that the government budget constraint holds in every period. The lump-sum transfer to old generations is financed printing money, so it does not enter the government budget constraint.

In real terms, the government budget constraint is given by

$$g_t = T_t + b_{i,t+1} - r_{t-1}b_{i,t} + b_{n,t+1} - r_{n,t}b_{n,t} \tag{11}$$

2.3.2 Monetary policy and long run inflation risk

In order to model long run inflation risk, it is assumed that government follows a money supply rule that implies base-level drift at a yearly horizon, similar to Hatcher (2014). In particular, the nominal money supply at the end of year j is given by[13]

$$M_j = M_{j-1}\Phi\exp(\varepsilon_j) = M_{j-N}\ \Phi^N \prod_{z=j-N+1}^{j}\exp(\varepsilon_z) \tag{12}$$

where $\Phi > 0$ and ε_j is an IID-normal money supply innovation with mean zero and standard deviation σ_M that hits the economy in year j.[14]

Since each period in the model lasts N years and the money supply is the end-of-year stock, Equation (12) implies that the nominal money supply in any period t is given by

$$M_t = \theta_t M_{t-1} = M_{t-1}\ \Phi^N \prod\nolimits_{j=1}^{N}\exp(\varepsilon_{j,t}) \tag{13}$$

where the money supply innovations have been indexed by $j = 1,2,..,N$ to indicate the year of period t in which they occur.

Notice that Equation (1) implies that $M_t/M_{t-1} = \Pi_t(c_{t,o}/c_{t-1,o})$. Hence, by Equation (13), inflation in period t is given by

$$\Pi_t = \frac{M_t/M_{t-1}}{c_{t,o}/c_{t-1,o}} = \frac{\theta_t}{c_{t,o}/c_{t-1,o}} = \frac{\Phi^N \prod\nolimits_{j=1}^{N}\exp(\varepsilon_{j,t})}{c_{t,o}/c_{t-1,o}} \tag{14}$$

Equation (14) shows that positive money supply innovations raise inflation, whereas positive productivity innovations lower inflation because they raise the marginal productivity of capital and hence push up retirement consumption. It is also clear from this equation that the money supply is a source of long run inflation risk.

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[13] The second equality in Equation (12) follows from repeated substitution for the past money supply N times.

[14] It is assumed for analytical convenience that money supply innovations are IID. If money supply innovations are positively correlated then the inflation risk premium is larger.

2.4 Market-clearing and equilibrium

Since capital depreciates fully in one period, investment in period t is $i_t = k_{t+1}$.

Definition of equilibrium:[15]

A set $\left\{c_{t,y}, c_{t,o}, b_{i,t}^d, b_{i,t}^s, b_{n,t}^d, b_{n,t}^s, k_t, m_t^d, m_t^s, g_t, R_t, r_t, r_{k,t}, w_t, T_t, TR_t, \Pi_t\right\}$ of allocations and prices with the following properties for all t:

(1) Allocations $\left\{c_{t,Y}, c_{t+1,O}, b_{i,t+1}^d, b_{n,t+1}^d, k_{t+1}\right\}$ solve the maximization problem of the generation born at time t and factors of production are paid their marginal products;

(2) The goods, money and bond markets clear:

$$
\begin{aligned}
y_t &= c_{t,Y} + c_{t,O} + g_t + k_{t+1} \\
m_t^d &= m_t^s \\
b_{i,t}^d &= b_{i,t}^s \\
b_{n,t}^d &= b_{n,t}^s
\end{aligned}
$$

(3) The government budget constraint is satisfied and spending is a constant fraction of GDP:

$$
\begin{aligned}
g_t &= T_t + b_{i,t+1}^s - r_{t-1} b_{i,t}^s + b_{n,t+1}^s - r_{n,t} b_{n,t}^s \\
g_t &= \phi_g y_t
\end{aligned}
$$

(4) The cash-in-advance constraint is binding: $m_t^d = \delta\, c_o$

3 The inflation risk premium: analytical results

Taking a second-order approximation of Equations (6) and (7) gives the following relationship between nominal and real interest rates:[16]

$$
\hat{R}_t = \hat{r}_t + E_t[\hat{\Pi}_{t+1}] - (1/2)(\phi_1 + 2\phi_2)\operatorname{var}_t[\hat{\Pi}_{t+1}] + \phi_3 \operatorname{cov}_t[\hat{c}_{t+1,.o}, \hat{\Pi}_{t+1}] + \operatorname{cov}_t[s\hat{d}f_{t+1.}, \hat{\Pi}_{t+1}] \quad (15)
$$

where 'hats' denote log deviations from the deterministic steady-state and

$$
\phi_1 \equiv \frac{\theta - \delta}{\theta + \delta(r\Pi - 1)}, \quad \phi_2 \equiv \frac{\delta}{\theta + \delta(r\Pi - 1)}, \quad \phi_3 \equiv \frac{\delta\,(r\Pi - 1)}{\theta + \delta(r\Pi - 1)}.
$$

The inflation risk premium is the final term in Equation (15), i.e. the covariance between the stochastic discount factor and inflation. It tells us that if inflation is high when the marginal utility of retirement consumption is high, then nominal debt will pay a higher equilibrium nominal interest rate to consumers to compensate for the fact that their real payoff will tend

[15] Notice that d and s superscripts have been introduced in this section to denote demand and supply values.

[16] See Appendix B for a derivation. Interestingly, Equation (15) shows that the relationship between real and nominal interest rates is non-standard: there is an additional covariance term involving consumption and inflation, and the coefficient on the inflation convexity term is no longer -1/2. The usual expression is recovered when $\delta = 0$. Mathematically, the standard expression is: $\hat{R}_t = \hat{r}_t + E_t[\hat{\Pi}_{t+1}] - (1/2)\operatorname{var}_t[\hat{\Pi}_{t+1}] + \operatorname{cov}_t[s\hat{d}f_{t+1.}, \hat{\Pi}_{t+1}]$.

to be low at times when extra consumption is highly valued. Since this covariance term is conditional on period-*t* information, it is only the component of marginal utility that is correlated with unanticipated inflation that matters for the inflation risk premium.[17]

The aim is to derive a second-order accurate closed-form solution for the inflation risk premium. As noted by Devereux and Sutherland (2011), this requires first-order accurate expressions for the stochastic discount factor and inflation. Log-linearizing the stochastic discount factor and Equation (14), we have:

$$\hat{sdf}_{t+1.} = -\gamma(\hat{c}_{t+1,.o} - \hat{c}_{t,y}) \text{ and } \hat{\Pi}_{t+1} = \sum_{j=1}^{N} \varepsilon_{j,t+1} - (\hat{c}_{t+1,o} - \hat{c}_{t,o}).$$

Inserting these results into the inflation risk premium term in Equation (15) gives

$$\begin{aligned} IRP &= \mathrm{cov}_t[\hat{sdf}_{t+1.}, \hat{\Pi}_{t+1}] = -\gamma\, \mathrm{cov}_t[\hat{c}_{t+1,.o}, \sum_{j=1}^{N} \varepsilon_{j,t+1} - \hat{c}_{t+1,o}] \\ &= \gamma\, \mathrm{var}_t[\hat{c}_{t+1,o}] - \gamma\, \mathrm{cov}_t[\hat{c}_{t+1,.o}, \sum_{j=1}^{N} \varepsilon_{j,t+1}] \end{aligned} \tag{16}$$

We therefore require an expression for old age consumption. Firstly, note that the money transfer and the cash-in-advance constraint, Equation (1), allow us to write Equation (3) as:

$$\begin{aligned} c_{t+1,o} &= r_{k,t+1}k_{t+1} + r_t b_{i,t+1} + r_{n,t+1} b_{n,t+1} + m_{t+1} \\ &= \frac{1}{1-\delta}(r_{k,t+1}k_{t+1} + [v_t r_t + (1-v_t) r_{n,t+1}] b_{t+1}) \end{aligned} \tag{17}$$

where $v_t \equiv b_{i,t+1} / b_{t+1}$ is the share of total government debt, $b_{t+1} = b_{i,t+1} + b_{n,t+1}$, that is indexed.

Log-linearizing the last equality in Equation (17) and using the deterministic steady-state as the approximation point for all variables except the indexation share v_t, and noting that $r_{n,t+1} = R_t / \Pi_{t+1}$ and $r = r_n$ at the deterministic steady-state:[18]

$$\begin{aligned} \hat{c}_{t+1,o} &= \frac{\theta_k}{1-\delta}[\hat{A}_{t+1} + \alpha \hat{k}_{t+1}] + \frac{\theta_b}{1-\delta}[v\hat{r}_t + (1-v)(\hat{R}_t - \hat{\Pi}_{t+1}) + \hat{b}_{t+1}] + \frac{\hat{v}_t b[r - r_n]}{1-\delta} \\ &= \frac{\theta_k}{1-\delta}[\hat{A}_{t+1} + \alpha \hat{k}_{t+1}] + \frac{\theta_b}{1-\delta}[v\hat{r}_t + (1-v)(\hat{R}_t - \hat{\Pi}_{t+1}) + \hat{b}_{t+1}] \end{aligned} \tag{18}$$

where $\theta_b \equiv rb/c_o$, $\theta_k \equiv 1 - \delta - \theta_b$, and $0 \le v \le 1$ is the approximation point around which the indexation share is log-linearised.

[17] Consequently, ρ_A and the anticipated part of inflation, $c_{t,o}$,will not matter for the inflation risk premium. Notice that the inflation risk premium is the only genuine risk premium in Equation (15) because the other covariance term is non-zero even if households are risk-neutral (i.e. when $\gamma = 0$).

[18] The indexation share is not approximated around the deterministic steady-state because the latter does not depend on *v*. Note that because the time-varying component of the indexation share does not affect the first-order behaviour of old-age consumption, see Eq(18), a constant portfolio share $v_t = v$ will support an equilibrium. This is a general result for second-order solutions established by Devereux and Sutherland (2011).

Now, substituting for inflation in Equation (18) and collecting terms, we have an expression for old-age consumption in terms of exogenous and predetermined variables:

$$\hat{c}_{t+1,o} = \frac{\theta_k}{1-\theta_b(1-v)-\delta}[\hat{A}_{t+1} + \alpha\hat{k}_{t+1}] + \frac{\theta_b}{1-\theta_b(1-v)-\delta}[v\hat{r}_t + (1-v)(\hat{R}_t - \sum_{j=1}^{N}\varepsilon_{j,t+1} + \hat{c}_{t,O}) + \hat{b}_{t+1}] \quad (19)$$

where $\hat{A}_{t+1} = \rho_A \hat{A}_t + e_{A,t+1}$ by Equation (10).

Finally, substituting Equation (19) into Equation (16) and noting that money innovations are uncorrelated gives us a second-order-accurate expression for the inflation risk premium:

$$\begin{aligned} IRP &= \gamma\,\mathrm{var}_t[\hat{c}_{t+1,o}] - \gamma\,\mathrm{cov}_t[\hat{c}_{t+1,.o}, \sum_{j=1}^{N}\varepsilon_{j,t+1}] \\ &= \gamma\left(\frac{\theta_k}{1-\theta_b(1-v)-\delta}\right)^2 \sigma_A^2 + \frac{\gamma\theta_b(1-v)}{1-\theta_b(1-v)-\delta}\left(1+\frac{\theta_b(1-v)}{1-\theta_b(1-v)-\delta}\right)N\sigma_M^2 > 0 \end{aligned} \quad (20)$$

Since each period in the model lasts N years, the annualised inflation risk premium can be calculated by dividing the above equation by N.[19] It should be noted that the parameter N enters Equation (20) directly because base-level drift in the money supply is a source of long run inflation risk, as is clear from Equation (14).

Equation (20) shows that the inflation risk premium increases with the coefficient of relative risk aversion, productivity risk, and money supply volatility. In addition, it depends on the steady-state share of retirement consumption that is funded by nominal bonds, θ_b; the indexation share, v; [20] and the steady-state share of capital income in retirement consumption, $\theta_k = 1 - \theta_b - \delta$. Productivity risk and money supply volatility enter the inflation risk premium because both productivity and money supply innovations will tend to lower retirement consumption (and hence raise marginal utility) at times when inflation is unexpectedly high.

A positive money supply innovation raises inflation through two channels. First, there is a direct effect on inflation through a rise money supply growth, plus an indirect effect due to the fact that a positive money supply innovation lowers retirement consumption (see Equation (19)), which in turn raises inflation further by Equation (14). Because unanticipated inflation reduces the real value of nominal debt, it lowers retirement consumption and raises marginal utility. As a result, monetary volatility raises the inflation risk premium.

Now consider the impact of productivity innovations. If there is an unexpectedly low innovation to productivity, then the marginal product of capital will fall, pushing down retirement consumption. At the same time, inflation will rise by Equation (14). Since productivity innovations will tend to raise marginal utility at times when inflation is high, productivity risk also makes a positive contribution to the inflation risk premium.

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[19] Formally, division by N follows from the assumption that annual yield = (N-year yield)$^{1/N}$. This conversion is common in overlapping generations models – see e.g. Constantinides and Mehra (2002, p. 285).

[20] Hatcher (2014) has shown numerically that the inflation risk premium in an overlapping generations model depends on the indexation share.

It is clear from Equation (20) that the inflation risk premium depends crucially on the steady-state shares of retirement consumption funded by nominal bonds, $(1 - v)\theta_b$, and capital, $\theta_k = 1 - \theta_b - \delta$. The bond share θ_b has an ambiguous impact on the inflation risk premium because raising the portfolio share of bonds makes the old more vulnerable to unexpectedly high money supply innovations that lower the real value of nominal government debt, but it also implies a simultaneous fall in the portfolio share of capital, which reduces exposure to productivity innovations. It is also clear from Equation (20) that the cash-in-advance parameter δ matters for the inflation risk premium. This point is considered in Section 5.4.

4 The inflation risk premium: numerical results from a calibrated model

In this section, a numerical example is considered. In particular, the model is calibrated and the inflation risk premium is compared with existing theoretical and empirical studies. The results reported in this section are based on a second-order approximation of the model around the log steady-state, as computed using Dynare (see Adjemian et al., 2011).[21]

4.1 Bond supply rule

To emphasise the generality of the analytical results in Section 3, the dynamics of government debt have thus far been left unspecified. This was possible because Equation (20) shows that, in a second-order approximation, the inflation risk premium is affected by the steady-state share of bond income in retirement consumption, θ_b, but not by the dynamics of the bond supply. Since the aim in this section is to compute a meaningful model 'estimate' for the inflation risk premium, a bond supply rule is chosen that implies a plausible steady-state share of bond income in retirement consumption and plausible ratios of key variables.

In particular, the bond supply rule follows that in Hatcher (2014):

$$E_t[sdf_{t+1}] = \beta \qquad (21)$$

This rule implies perfect consumption smoothing at the deterministic steady-state. The deterministic steady-state of the model under this rule is reported in Appendix C.

4.2 Calibration

The model is roughly calibrated to the US economy. In particular, the parameters of the model are chosen to roughly match key ratios in the data. Expressions for these key ratios at the deterministic steady-state are reported in Appendix C. Since these ratios depend upon several different parameters, the calibration uses parameter values which are plausible *and* give good performance against target ratios.[22] The calibration is listed in Table 1.

[21] The results reported by Dynare in this section match the predictions of Equation (20) exactly.

[22] In some models, key ratios are pinned down by a single parameter so that calibrated values can be set to replicate target ratios exactly. The model here does not have this property.

Table 1 – Baseline calibrated values

α	Capital share	0.22	θ_g	Government spending-GDP ratio	0.16
β	Private discount factor	2/3	Φ	Annual steady-state money growth	1.02
γ	Risk aversion	5	ρ_A	Productivity persistence	0.50
N	Number of years per period	30	σ_A	Std(TFP innov.)	0.07
δ	Cash-in-advance parameter	0.05	σ	Std (money innov.)	0.02
v	Share of indexed debt	0.10			

The number of years per period N is set equal to 30, implying that the bonds have a maturity of 30 years. Steady-state annual money growth Φ is set at 1.02, consistent with steady-state inflation of 2%. The parameter θ_g is set at 0.16 to match a government spending-GDP share of 16%.[23] The production function parameter α is set at 0.22, implying that capital income is 22% of GDP. This value is on the low side of standard calibrations, but this helps the model to match the target ratios of long-term debt and investment to GDP (see Table 2 and Appendix C). The cash-in-advance parameter δ is set at 0.05, which implies that 5% of retirement consumption is funded by money. This calibration implies that cash holdings are 2% of GDP, which is fairly similar to the currency component of M1 in US data.[24]

Since real returns are equalised in the absence of uncertainty, the share of indexed debt v does not affect the deterministic steady-state (see Appendix C). It is therefore set at 0.1 to match the 2008 share of outstanding marketable debt in Treasury Inflation-Protected Securities (see Campbell et al., 2009).[25] The discount factor β is set at 2/3, which implies an annual discount factor of 0.987 and a steady-state real interest rate of around 1.5% per annum. Since inflation is 2% per year in the steady-state, the nominal rate is 3.5% per annum. The coefficient of relative risk aversion γ is set at 5. This value is on the high side of standard calibrations to give the model a chance of matching empirical estimates of the inflation risk premium.

The productivity innovation standard deviation σ_A is set at 0.07 and productivity persistence ρ_A at 0.5. Both of these calibrated values are similar to those in Hatcher (2014), who considers a model with a generational horizon of 20 years.[26] Finally, the standard deviation of money supply innovations is set at 0.02, which lies slightly below the standard deviation of annual M2 growth from 1992 to 2007 based on FRED data.

4.3 Results and discussion

The calibrated model does fairly well against target ratios (see Table 2). The US investment-GDP ratio has been close to 15% over the period 1992 to 2012 and over the same period the consumption share averaged 69% (see Table 1, 2013 Economic Report of the President), implying target ratios of 0.15 and 0.69. The model gives ratios of 0.14 and 0.70. The US government debt to GDP share averaged around 70 per cent over the same period and, based

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[23] The average government spending share over the period 1992 to 2012 was approximately 16%. See Economic Report of the President 2013, Table 1.

[24] See the data available from the FRED database run by the Federal Reserve Bank of St Louis.

[25] Note that the indexation share can be treated as a calibrated parameter because (i) the returns R and r are endogenous, and (ii) we established (see Equation (18) and Footnote 18) that, in general, a constant indexation share supports an equilibrium up to a second-order approximation of the model.

[26] The higher value of σ_A here (0.070 vs 0.055) is chosen to reflect the longer generational horizon of 30 years.

on OECD data, bonds with maturities exceeding 20 years but less than or equal to 30 years accounted for one-tenth of all bonds outstanding in 2002.[27] Multiplying these two figures together implies a target share of long-term debt to GDP of 7%. On this score, the model gives a ratio of 0.07. Finally, the model implies that income from nominal bonds accounts for 29% of retirement consumption at the deterministic steady-state. This figure is fairly similar to the net nominal positions of US households aged 66-75 and over 75 as a fraction of average net worth: 19.4% and 30.4% respectively in 1989 (see Doepke and Schneider, 2006).

The inflation risk premium in the calibrated model is non-trivial at 20 basis points per annum. In fact, this figure is comparable to some recent empirical estimates from estimated no-arbitrage affine models of the term structure. For instance, in relation to the Euro Area, Garcia and Werner (2010) estimate an inflation risk premium at a 5-year maturity of around 25 basis points, while Hördahl and Tristani (2012) report an average risk premium at the 10-year maturity of just over 20 basis points. Most empirical estimates from affine models of the term structure tend to be somewhat higher than these figures, however.

Table 2 – Inflation risk premium and target versus model ratios

	Target	Definition	Deterministic	Stochastic
b/y	0.07	Long-term bonds/GDP	0.07	0.07
i/y	0.14	Investment/GDP	0.14	0.14
$(c_y + c_o)/y$	0.69	Consumption/GDP	0.70	0.70
$(1-v)\theta_b$	0.25	$\dfrac{\text{Nominal bond income}}{\text{Retirement consumption}}$	0.29	0.29
IRP	–	Inflation risk premium (p.a.)	0	20.01

Note: the value of g/y is given by $1 - (c_y + c_o)/y - i/y$. The IRP is in basis points per annum.

In the US, several recent studies have estimated no-arbitrage affine models of the term structure using nominal yields and inflation data, along with additional information from index-linked yields, inflation surveys or inflation swaps. These studies point to a positive average inflation risk premium of between 50 and 120 basis points. For instance, D'Amico et al. (2009) found an inflation risk premium on 10-year bonds of 64 basis points on average, while Ang et al. (2008) report a higher average risk premium of around 110 basis points at a 10-year maturity. Even higher estimates at a 10-year maturity have been reported by Chernov and Mueller (2012). Estimates of inflation risk-premia on long-term US bonds have been provided by Haubrich et al. (2008). Their results suggest that inflation risk premia increase with maturity: 10-year bonds have a risk premium of 51 basis points, compared to 81 basis points on 20-year bonds, and 101 basis points on 30-year bonds. To the author's knowledge, this is the only study estimating long-term inflation risk premia. While this study concludes that long-term inflation risk premia are somewhat higher than in the calibrated model, the model's prediction is of the same order of magnitude.

[27] The data on the government debt-GDP share was obtained from the FRED database.

There are fewer empirical estimates of inflation risk premia for the UK, but there have been some notable recent attempts. Joyce et al. (2010) estimated a no-arbitrage affine model of the term structure and concluded that the inflation risk premium on 5-year bonds has averaged around 100 basis points. More recently, Andreasen (2012) estimated a New Keynesian model of the UK economy and found an average inflation risk premium of around 50 basis points on 5-year bonds and 70 basis points on 10-year bonds. The results in Reschreiter (2004) likewise suggest that the UK inflation risk premium is non-trivial at these (and longer) maturities.

Turning to the theoretical literature, general equilibrium models have had a difficult time explaining the magnitude of inflation risk premia, just as with other asset risk premia.[28] For instance, Ravenna and Seppälä (2007) simulated a New Keynesian model of the term structure and found that inflation risk premia were robustly small and negative at bond maturities up to five years. De Paoli et al. (2010) showed that the sign of the inflation risk premium depends crucially on shock composition: it is positive in a New Keynesian world dominated by productivity shocks but negative when monetary policy shocks are dominant. The positive inflation risk premium in their model amounts to 9 basis points on a 1-quarter bond. In a similar New Keynesian model, Hördahl et al. (2008) find very small positive inflation risk premia of less than 1 basis point at maturities from 1 to 10 years. Lastly, as noted above, Andreasen (2012) estimated a New Keynesian model of the term structure on UK data and found an average inflation risk premium on 5-year bonds of around 50 basis points. However, this figure relies crucially on high estimated risk aversion through Epstein-Zin preferences. Consequently, the model would not have been able to generate high risk premia if plausible levels of risk aversion had been imposed on the model in estimation (e.g. by placing bounds on the estimated preference parameters related to risk aversion).

In summary, the empirical literature points to robustly positive inflation risk premia in major developed economies, with large premia in the US and the UK. There are few studies investigating long-term premia, but the available evidence points to an inflation risk premium of around 80 basis points on 20-year bonds and 100 basis points on 30-year bonds. Although the risk premium of 20 basis points in the calibrated model is somewhat lower than these figures, it is of the same order of magnitude as most empirical estimates in the literature.

5 Robustness

The analysis of the previous section shows that the inflation risk premium is non-trivial in a calibrated model. This section discusses the role of the cash-in-advance constraint in this result and investigates the sensitivity of the risk premium to key calibrated parameters.

5.1 Importance of the cash-in-advance constraint

There are two ways to assess the importance of the cash-in-advance constraint for the inflation risk premium. First, changing the cash-in-advance parameter δ changes the fraction of old-age consumption that must be purchased using money and alters the composition of retirement portfolios. Second, we can ask how the inflation risk premium from the calibrated model compares with that in a model with no cash-in-advance motive.

[28] The difficulties that theoretical models face in matching risk-premia are well-known and apply to both bonds (e.g. Backus et al., 1989; Rudebsuch and Swanson, 2008) and equity (e.g. Mehra and Prescott, 1985).

5.1.1 Cash-in-advance parameter

As is clear from Equation (20), the cash-in-advance parameter δ enters the inflation risk premium directly, with this direct effect being ambiguous. It will also enter this expression indirectly through the coefficient $\theta_b = rb/c_o$ (the steady-state share of bond income in retirement consumption). In the case of the calibrated model, it can be shown using the expressions in Appendix C that a rise in δ will tend to reduce θ_b under standard parameter values. Since δ has an ambiguous impact on the inflation risk premium, we resort to numerical analysis by simulating the inflation risk premium under alternative values of δ.

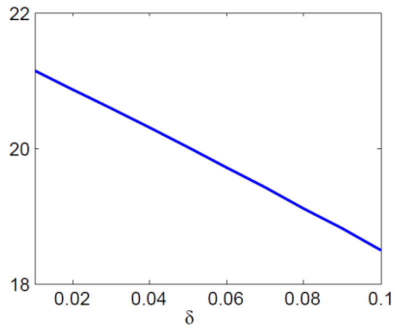

Fig 1 – Sensitivity of the inflation risk premium to the cash-in-advance parameter, δ.
Note: The inflation risk premium is in basis points per annum. The baseline case is δ = 0.05.

Figure 1 shows that the inflation risk premium is relatively insensitive to δ. For instance, doubling the value of δ from the baseline value of 0.05 to 0.10 lowers the inflation risk premium to 18 basis points, whereas reducing it to 0.01 raises the inflation risk premium to 21 basis points. Although Figure 1 seems to suggest that setting $\delta = 0$ would raise the inflation risk premium, this is not case because money (and hence inflation) is absent. Using such a benchmark is clearly not sensible because it implies zero inflation risk premium by definition. Instead, it is more appropriate to compare against a case where money is valued but the cash-in-advance motive is absent. We now turn to a comparison of this kind.

5.1.2 Absence of the cash-in-advance motive

Champ and Freeman (1990) consider an overlapping generations model in which money is introduced by a legal requirement to hold real money balances of at least $\mu > 0$, so that $m_t = \mu$ for all t when the constraint is binding. As a result, money has value in equilibrium, but the cash-in-advance motive is absent. To assess the implications of this, an analytical expression the inflation risk premium was derived under this alternative assumption (see Supplementary Appendix). The inflation risk premium is now given by

$$IRP = \gamma\theta_b(1-v)N\sigma_M^2 \qquad (22)$$

where, as previously, $\theta_b \equiv rb/c_o$ is the steady-state share of bond income .

If this alternative model is calibrated with the same values of γ, v, N, and σ_M as the baseline model and the production function parameter α is chosen to give the same bond share θ_b, then the predicted inflation risk premium is much lower at only 5.4 basis points per annum. This finding shows that the cash-in-advance motive is crucial in producing a sizeable risk premium. In particular, it raises the inflation risk premium by almost 15 basis points per annum and more than trebles its value. It is also notable that productivity risk does not matter for the inflation risk premium in the absence of the cash-in-advance motive.

The cash-in-advance constraint is important for the inflation risk premium for two reasons. First, it implies that inflation is positively related to money supply growth but inversely related to retirement consumption; see Equation (14). Consequently, the positive correlation between stochastic discount factor and inflation is stronger than in the absence of the cash-in-advance motive. Second, because inflation depends on monetary growth and retirement consumption, inflation risk depends on both productivity risk and monetary volatility. By contrast, all inflation risk comes from monetary volatility in the absence of a cash-in-advance motive (see the Supplementary Appendix). As a result, inflation risk is higher for any given level of volatility in the cash-in-advance model.

5.2 Parameter sensitivity analysis

This section considers robustness to several key calibrated parameters, namely, the coefficient of relative risk aversion γ, the indexation share v, monetary risk σ_M and productivity risk σ_A. As shown in Appendix C, these parameters do not affect the model's deterministic steady-state. Consequently, they can be varied to test robustness while maintaining the deterministic steady-state listed in Table 1. The results are shown in Figure 2.

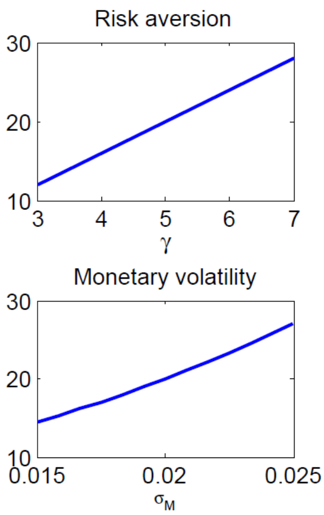

Fig 2 – Sensitivity of the inflation risk premium to key parameters. Note: the inflation risk premium is in basis points per annum.

With the exception of productivity risk, the inflation risk premium is quite sensitive to these parameters. Under high risk aversion of 7 the inflation risk premium rises to almost 30 basis points, while under a standard parameterization such as 3 it falls to around 12 basis points. Consistent with Equation (20), the inflation risk premium falls at a linear rate as the indexation share v is increased. It falls from 24 to 17 basis points as the indexation share is increased from 0 to 0.20, which appears to be the most relevant range for major developed economies.[29] For monetary volatility, there is also considerable sensitivity: raising the

[29] For instance, in Canada in 2008, indexed bonds were around 6% of marketable government debt (Department of Finance Canada, 2008). Similarly, Campbell et al. (2009) report that, in 2008, indexed government bonds were 10% of marketable government debt in the US. In the UK, which has the highest indexation share amongst developed economies, the indexation share has been between 20% and 30% over the past decade.

monetary innovation standard deviation to 0.025 raises the inflation risk premium to almost 30 basis points, while lowering it to 0.015 gives an inflation risk premium of around 15 basis points. By contrast, raising the standard deviation of productivity innovations from 0.055 to 0.085 raises the inflation risk premium only modestly, from 17 to 23.5 basis points. All in all, the conclusion that the model produces a non-trivial inflation risk premium is fairly robust, with the inflation risk premium not falling below 12 basis points.

6 Conclusion

This paper has investigated the inflation risk premium on long-term government debt in an overlapping generations model. In order to capture long run inflation risk, money supply shocks hit the economy at a yearly horizon and lead to base-level drift. The paper makes two main contributions. First, using the methods developed by Lombardo and Sutherland (2007) and Devereux and Sutherland (2011), a second-order accurate closed-form analytical expression for the inflation risk premium was derived. This expression shows that the risk premium depends crucially on the importance of nominal bonds and capital as sources of retirement consumption. This finding is appealing because the data suggests that an important reason old households are hit by unanticipated inflation is their substantial holdings of nominal government debt (Doepke and Schneider, 2006; Meh and Terajima, 2011).

The second contribution was to show that the inflation risk premium predicted by the model is non-trivial under plausible calibrations of risk aversion. In particular, when the model was roughly calibrated to US data it gave an inflation risk premium of around 20 basis points. This number is of the same order of magnitude as most empirical estimates and fairly robust in sensitivity analysis. In the model presented, money was introduced by a cash-in-advance constraint on old-age consumption. This assumption is crucial because it raises inflation risk and strengthens the correlation between inflation and the stochastic discount factor. In fact, the inflation risk premium is lowered to only 5 basis points in the same model if the cash-in-advance motive is absent.

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Appendix A – Derivation of first-order conditions

This Appendix derives the first-order conditions for households reported in the main text.

To start with, note that the budget constraints, Equations (2) and (3), can be written as follows where the lump-sum monetary transfer to the old is taken as given by the old:

$$c_{t,y} = w_t - T_t - x_{t+1} - m_t \tag{A1}$$

$$c_{t+1,o} = r_{x,t+1}x_{t+1} + r_{m,t+1}m_t + TR_{real,t+1} \tag{A2}$$

where $x_{t+1} \equiv k_{t+1} + b_{i,t+1} + b_{n,t+1}$ and $r_{x,t+1} \equiv \lambda_{k,t} r_{k,t+1} + \lambda_{n,t} r_{n,t+1} + \lambda_{i,t} r_t$ with shares $\lambda_{k,t} \equiv k_{t+1} / x_{t+1}$, $\lambda_{n,t} \equiv b_{n,t+1} / x_{t+1}$ and $\lambda_{i,t} = 1 - \lambda_{k,t} - \lambda_{n,t}$.

Rearranging for x_{t+1} in (A2) and substituting the result into (A1) gives us the lifetime budget constraint faced by the young born in period t:

$$c_{t,y} + \frac{c_{t+1,o}}{r_{x,t+1}} = w_t + \frac{TR_{real,t+1}}{r_{x,t+1}} - \left(\frac{\Pi_{t+1} r_{x,t+1} - 1}{\Pi_{t+1} r_{x,t+1}} \right) m_t \tag{A3}$$

where the fact that $r_{m,t+1} = 1/\Pi_{t+1}$ has been used.

Due to the cash-in-advance constraint, $m_{t+1} = \delta c_{t+1,o}$, the last term in (A3) will be a function of consumption in old age. In particular, given that $\Pi_{t+1} \equiv P_{t+1}/ P_t$ and $M_{t+1} = \theta_{t+1} M_t$:

$$\begin{aligned} \left(\frac{\Pi_{t+1} r_{x,t+1} - 1}{\Pi_{t+1} r_{x,t+1}} \right) m_t &= \left(\frac{\Pi_{t+1} r_{x,t+1} - 1}{\Pi_{t+1} r_{x,t+1}} \right) \frac{M_t}{P_t} = \left(\frac{\Pi_{t+1} r_{x,t+1} - 1}{\Pi_{t+1} r_{x,t+1}} \right) \frac{\Pi_{t+1}}{\theta_{t+1}} m_{t+1} \\ &= (\Pi_{t+1} r_{x,t+1} - 1) \frac{\delta c_{t+1,o}}{r_{x,t+1} \theta_{t+1}} \end{aligned} \tag{A4}$$

Substituting for (A4) in (A3) and collecting terms gives

$$c_{t,y} + \left(1 + \frac{\delta(\Pi_{t+1} r_{x,t+1} - 1)}{\theta_{t+1}} \right) \frac{c_{t+1,o}}{r_{x,t+1}} = w_t + \frac{TR_{real,t+1}}{r_{x,t+1}} \tag{A5}$$

The maximization problem of the young born in period t can therefore be stated as follows:

$$\max_{\{c_{t,y}, c_{t+1,O}\}} U_t = \frac{c_{t,y}^{1-\gamma} - 1}{1-\gamma} + \beta E_t \left[\frac{c_{t+1,o}^{1-\gamma} - 1}{1-\gamma} \right] \tag{A6}$$

$$\text{s.t} \quad c_{t,y} + \left(1 + \frac{\delta(\Pi_{t+1} r_{x,t+1} - 1)}{\theta_{t+1}} \right) \frac{c_{t+1,o}}{r_{x,t+1}} = w_t + \frac{TR_{real,t+1}}{r_{x,t+1}}$$

The first-order condition for this problem is as follows:

$$1 = E_t \left[\frac{sdf_{t+1} r_{x,t+1}}{1 + \delta(\Pi_{t+1} r_{x,t+1} - 1)/\theta_{t+1}} \right] \tag{A7}$$

where $sdf_{t+1} \equiv \beta(c_{t+1,o} / c_{t,y})^{-\gamma}$.

The Euler equations for individual asset holdings (see Section 2.1) can be obtained from (A7) by setting $\lambda_{j,t} = 1$ for $j \in \{k, n, i\}$ in the definition of $r_{x,t+1}$.

Appendix B – The relationship between real and nominal interest rates

In this Appendix, the second-order accurate approximation method in Lombardo and Sutherland (2007) is used to derive the relationship between real and nominal rates. This method is applied to portfolio problems by Devereux and Sutherland (2011), amongst others.

For convenience, the first-order conditions for bond holdings are repeated here:

$$1 = E_t[sdf_{t+1} \tilde{r}_{n,t+1}] \quad \text{for nominal bonds, } b_n \tag{B1}$$

$$1 = E_t[sdf_{t+1} \tilde{r}_{i,t+1}] \quad \text{for indexed bonds, } b_i \tag{B2}$$

where $sdf_{t+1} \equiv \beta(c_{t+1,o} / c_{t,y})^{-\gamma}$, $\tilde{r}_{n,t+1} \equiv r_{n,t+1}[1 + \delta(r_{n,t+1}\Pi_{t+1} - 1)/\theta_{t+1}]^{-1}$, $r_{n,t+1} = R_t / \Pi_{t+1}$ and $\tilde{r}_{i,t+1} \equiv r_t[1 + \delta(r_t \Pi_{t+1} - 1)/\theta_{t+1}]^{-1}$.

We start by taking a second-order approximation of (B1) and (B2):

$$0 = E_t[s\hat{d}f_{t+1} + (1/2)s\hat{d}f_{t+1}^2 + \hat{\tilde{r}}_{n,t+1} + (1/2)\hat{\tilde{r}}_{n,t+1}^2 + s\hat{d}f_{t+1}\hat{\tilde{r}}_{n,t+1}] + O[2] \tag{B3}$$

$$0 = E_t[s\hat{d}f_{t+1} + (1/2)s\hat{d}f_{t+1}^2 + \hat{\tilde{r}}_{i,t+1} + (1/2)\hat{\tilde{r}}_{i,t+1}^2 + s\hat{d}f_{t+1}\hat{\tilde{r}}_{i,t+1}] + O[2] \tag{B4}$$

where 'hats' denote log deviations from the deterministic steady-state and $O[2]$ includes all terms of higher order than 2.

Subtracting (B4) from (B3) and ignoring higher-order terms, we get:

$$0 = E_t[\hat{\tilde{r}}_{n,t+1} - \hat{\tilde{r}}_{i,t+1} + (1/2)(\hat{\tilde{r}}_{n,t+1}^2 - \hat{\tilde{r}}_{i,t+1}^2) + s\hat{d}f_{t+1}(\hat{\tilde{r}}_{n,t+1} - \hat{\tilde{r}}_{i,t+1})] \tag{B5}$$

As noted by Hördahl et al. (2008) and Devereux and Sutherland (2011), second-order accurate expressions for asset prices can be obtained from first-order accurate solutions. In particular, terms involving *products* can be computed using their log-linear counterparts.[30]

To this end, note that the returns that enter Equations (B1) and (B2) can be approximated to first-order around the deterministic steady-state as follows:

[30] See Proposition 2 in Devereux and Sutherland (2011).

$$\hat{\hat{r}}_{n,t+1} = \frac{\theta-\delta}{\theta+\delta(r\Pi-1)}\hat{r}_{n,t+1} - \frac{\delta\, r\Pi}{\theta+\delta(r\Pi-1)}\hat{\Pi}_{t+1} + \frac{\delta\,(r\Pi-1)}{\theta+\delta(r\Pi-1)}\hat{\theta}_{t+1} \quad \text{(B6)}$$

$$\hat{\hat{r}}_{i,t+1} = \frac{\theta-\delta}{\theta+\delta(r\Pi-1)}\hat{r}_{t} - \frac{\delta\Pi r}{\theta+\delta(r\Pi-1)}\hat{\Pi}_{t+1} + \frac{\delta\,(r\Pi-1)}{\theta+\delta(r\Pi-1)}\hat{\theta}_{t+1} \quad \text{(B7)}$$

Noting that $\hat{\theta}_{t+1} = \hat{\Pi}_{t+1} + \hat{c}_{t+1,o} - \hat{c}_{t,o}$ and $\hat{r}_{n,t+1} = \hat{R}_t - \hat{\Pi}_{t+1}$, we can simplify these equations to

$$\hat{\hat{r}}_{n,t+1} = \phi_1\hat{R}_t - (\phi_1+\phi_2)\hat{\Pi}_{t+1} + \phi_3(\hat{c}_{t+1,o} - \hat{c}_{t,o}) \quad \text{(B8)}$$

$$\hat{\hat{r}}_{i,t+1} = \phi_1\hat{r}_t - \phi_2\hat{\Pi}_{t+1} + \phi_3(\hat{c}_{t+1,o} - \hat{c}_{t,o}) \quad \text{(B9)}$$

where $\phi_1 \equiv \frac{\theta-\delta}{\theta+\delta(r\Pi-1)}$, $\phi_2 \equiv \frac{\delta}{\theta+\delta(r\Pi-1)}$, and $\phi_3 \equiv \frac{\delta\,(r\Pi-1)}{\theta+\delta(r\Pi-1)}$.

Due to certainty equivalence, the log-linearized returns in (B8) and (B9) must satisfy:

$$E_t[\hat{\hat{r}}_{n,t+1}] = E_t[\hat{\hat{r}}_{i,t+1}], \quad \text{(B10)}$$

This observation simplifies somewhat the derivations which follow. First, taking expectations in (B5) and using (B10) in terms that involve *products* gives a second-order accurate expression for the expected difference in real returns involving second moments only:[31]

$$\begin{aligned} E_t[\hat{\hat{r}}_{n,t+1} - \hat{\hat{r}}_{i,t+1}] &= -\frac{1}{2}(E_t[\hat{\hat{r}}^2_{n,t+1}] - E_t[\hat{\hat{r}}^2_{i,t+1}]) - (E_t[s\hat{d}f_{t+1}\hat{\hat{r}}_{n,t+1}] - E_t[s\hat{d}f_{t+1}\hat{\hat{r}}_{n,t+1}]) \\ &= -\frac{1}{2}(\mathrm{var}_t[\hat{\hat{r}}_{n,t+1}] - \mathrm{var}_t[\hat{\hat{r}}_{i,t+1}]) - (\mathrm{cov}_t[s\hat{d}f_{t+1},\hat{\hat{r}}_{n,t+1}] - \mathrm{cov}_t[s\hat{d}f_{t+1},\hat{\hat{r}}_{i,t+1}]) \end{aligned} \quad \text{(B11)}$$

Using (B8) and (B9) to compute the terms on the RHS of (B11), we have:

$$\mathrm{var}_t[\hat{\hat{r}}_{n,t+1}] = (\phi_1+\phi_2)^2\,\mathrm{var}_t[\hat{\Pi}_{t+1}] + \phi_3^2\,\mathrm{var}_t[\hat{c}_{t+1,o}] - 2(\phi_1+\phi_2)\phi_3\,\mathrm{cov}_t[\hat{c}_{t+1,o},\hat{\Pi}_{t+1}] \quad \text{(B12)}$$

$$\mathrm{var}_t[\hat{\hat{r}}_{i,t+1}] = \phi_2^2\,\mathrm{var}_t[\hat{\Pi}_{t+1}] + \phi_3^2\,\mathrm{var}_t[\hat{c}_{t+1,o}] - 2\phi_2\phi_3\,\mathrm{cov}_t[\hat{c}_{t+1,o},\hat{\Pi}_{t+1}] \quad \text{(B13)}$$

$$\begin{aligned} \mathrm{cov}_t[s\hat{d}f_{t+1},\hat{\hat{r}}_{n,t+1}] &= \mathrm{cov}_t[s\hat{d}f_{t+1}, -(\phi_1+\phi_2)\hat{\Pi}_{t+1} + \phi_3\hat{c}_{t+1,o}] \\ &= -\gamma\phi_3\,\mathrm{var}_t[\hat{c}_{t+1,o}] - (\phi_1+\phi_2)\,\mathrm{cov}_t[s\hat{d}f_{t+1},\hat{\Pi}_{t+1}] \end{aligned} \quad \text{(B14)}$$

$$\begin{aligned} \mathrm{cov}_t[s\hat{d}f_{t+1},\hat{\hat{r}}_{i,t+1}] &= \mathrm{cov}_t[s\hat{d}f_{t+1}, -\phi_2\hat{\Pi}_{t+1} + \phi_3\hat{c}_{t+1,o}] \\ &= -\gamma\phi_3\,\mathrm{var}_t[\hat{c}_{t+1,o}] - \phi_2\,\mathrm{cov}_t[s\hat{d}f_{t+1},\hat{\Pi}_{t+1}] \end{aligned} \quad \text{(B15)}$$

[31] The second equality uses $E_t[z^2_{t+1}] = \mathrm{var}_t[z_{t+1}] + (E_t[z_{t+1}])^2$ and $E_t[z_{t+1}s_{t+1}] = \mathrm{cov}_t[z_{t+1},s_{t+1}] + E_t[z_{t+1}]E_t[s_{t+1}]$ along with equalization of expected real returns as per (B10).

Substituting these expressions into the second line of (B11) implies that

$$E_t[\hat{\tilde{r}}_{n,t+1}-\hat{\tilde{r}}_{i,t+1}]=-\frac{1}{2}[\,(\phi_1^2+2\phi_1\phi_2)\operatorname{var}_t[\hat{\Pi}_{t+1}]-2\phi_1\phi_3\operatorname{cov}_t[\hat{c}_{t+1,o},\hat{\Pi}_{t+1}]\,]+\phi_1\operatorname{cov}_t[s\hat{d}f_{t+1},\hat{\Pi}_{t+1}] \quad \text{(B16)}$$

Finally, by (B8) and (B9), the left hand side of (B16) is equal to

$$E_t[\hat{\tilde{r}}_{n,t+1}-\hat{\tilde{r}}_{i,t+1}]=\phi_1(\hat{R}_t-\hat{r}_t)-\phi_1 E_t[\hat{\Pi}_{t+1}] \quad \text{(B17)}$$

Substituting (B17) into (B16) and dividing by ϕ_1 gives us the equation reported in Section 3:

$$\hat{R}_t=\hat{r}_t+E_t[\hat{\Pi}_{t+1}]-(1/2)(\phi_1+2\phi_2)\operatorname{var}_t[\hat{\Pi}_{t+1}]+\phi_3\operatorname{cov}_t[\hat{c}_{t+1,.o},\hat{\Pi}_{t+1}]+\operatorname{cov}_t[s\hat{d}f_{t+1.},\hat{\Pi}_{t+1}] \quad \text{(B18)}$$

Appendix C – Deterministic steady state of the calibrated model and key ratios

Deterministic steady-state

$$c_y=(1-\alpha)k^{\alpha}-T-b-k-m \quad \text{(C1)}$$

$$c_o=\alpha k^{\alpha}+rb+r_m m+TR_{real}=(\alpha k^{\alpha}+rb)/(1-\delta) \quad \text{(C2)}$$

$$TR_{real}=(1-r_m)m \quad \text{(C3)}$$

$$r_m=1/\Pi \quad \text{(C4)}$$

$$m=\delta c_o \quad \text{(C5)}$$

$$\Pi=\theta=\Phi^N \quad \text{(C6)}$$

$$\tilde{r}_i=1/\beta \quad \text{(C7)}$$

$$\tilde{r}_n=\tilde{r}_k=\tilde{r}_i \quad \text{(C8)}$$

$$R=\Pi\, r \quad \text{(C9)}$$

$$A=1 \quad \text{(C10)}$$

$$r_k=\alpha k^{\alpha-1}=r=\frac{\theta-\delta}{\theta(\beta-\delta)} \quad \text{(C11)}$$

$$r_n=r \quad \text{(C12)}$$

$$w=(1-\alpha)k^{\alpha} \quad \text{(C13)}$$

$$i=k=(\alpha/r)^{1/(1-\alpha)} \quad \text{(C14)}$$

$$y=k^{\alpha}=(\alpha/r)^{\alpha/(1-\alpha)} \quad \text{(C15)}$$

$$g=\phi_g y \quad \text{(C16)}$$

$$T=g-(1-r)b \quad \text{(C17)}$$

$$b=\frac{(1-\alpha-(1+\delta)\alpha/(1-\delta)-\phi_g)k^{\alpha}-k}{r/(1-\delta)} \quad \text{(C18)}$$

$b_i=vb=b-b_n$ (implying that $b^n=(1-v)b$) (C19)

Deterministic steady-state ratios

$$\frac{m}{y} = \delta \frac{c_o}{y} = \frac{\delta\alpha}{1-\delta} + \frac{\delta r}{1-\delta}\frac{b}{y} \tag{C20}$$

$$\frac{i}{y} = \frac{k}{y} = (\alpha / r) \tag{C21}$$

$$\frac{g}{y} = \theta_g \tag{C22}$$

$$\frac{c}{y} = \frac{c_y + c_o}{y} = (1-\alpha+(1+\delta)\alpha/(1-\delta)-\phi_g) - \frac{r}{1-\delta}\frac{b}{y} \tag{C23}$$

$$\frac{b}{y} = \frac{1}{r/(1-\delta)}\left((1-\alpha-(1+\delta)\alpha/(1-\delta)-\phi_g) - \frac{i}{y}\right) \tag{C24}$$

$$\theta_b = \frac{rb}{c_o} = \frac{(1-\delta)(1-\alpha-(1+\delta)\alpha/(1-\delta)-\phi_g)k^{\alpha}-k}{(1-\alpha/(1-\delta)-\phi_g)k^{\alpha}-k} \tag{C25}$$

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Supplementary Appendix (for online publication only)

The inflation risk premium in the absence of a cash-in-advance constraint

In this Appendix the inflation risk premium is derived in the case where, following Champ and Freeman (1990), young agents are required to hold real money balances of at least $\mu > 0$, so that $m_t \geq \mu$. As in that paper, this constraint is assumed to bind, so that $m_t = \mu$ for all t. All other aspects of the model are unchanged. This change in assumption affects household first-order conditions and the equations for inflation and consumption by the old. The first-order conditions are derived and the implications for inflation and consumption by the old are discussed. Finally, we turn to the inflation risk premium.

First-order conditions

We start by noting that the budget constraints can be written as follows where the lump-sum monetary transfer to the old is taken as given:

$$c_{t,y} = w_t - T_t - x_{t+1} - m_t \tag{SA1}$$

$$c_{t+1,o} = r_{x,t+1} x_{t+1} + r_{m,t+1} m_t + TR_{real,t+1} \tag{SA2}$$

where $x_{t+1} \equiv k_{t+1} + b_{i,t+1} + b_{n,t+1}$ and $r_{x,t+1} \equiv \lambda_{k,t} r_{k,t+1} + \lambda_{n,t} r_{n,t+1} + \lambda_{i,t} r_t$ with $\lambda_{k,t} \equiv k_{t+1} / x_{t+1}, \lambda_{n,t} \equiv b_{n,t+1} / x_{t+1}$ and $\lambda_{i,t} = 1 - \lambda_{k,t} - \lambda_{n,t}$.

Rearranging for x_{t+1} in (SA2) and substituting the result into (SA1) gives us the lifetime budget constraint of the young born in period t:

$$c_{t,y} + \frac{c_{t+1,o}}{r_{x,t+1}} = w_t + \frac{TR_{real,t+1}}{r_{x,t+1}} - \left(\frac{\Pi_{t+1} r_{x,t+1} - 1}{\Pi_{t+1} r_{x,t+1}} \right) m_t = w_t + \frac{TR_{real,t+1}}{r_{x,t+1}} - \left(\frac{\Pi_{t+1} r_{x,t+1} - 1}{\Pi_{t+1} r_{x,t+1}} \right) \mu \tag{SA3}$$

where $r_{m,t+1} = 1/\Pi_{t+1}$ and $m_t = \mu$ have been used.

Hence, the maximization problem of the young born in period t is as follows:

$$\max_{\{c_{t,y},c_{t+1,O}\}} U_t = \frac{c_{t,y}^{1-\gamma}-1}{1-\gamma} + \beta E_t\left[\frac{c_{t+1,o}^{1-\gamma}-1}{1-\gamma}\right] \tag{SA4}$$

$$\text{s.t} \quad c_{t,y} + -\frac{c_{t+1,o}}{r_{x,t+1}} = w_t + \frac{TR_{real,t+1}}{r_{x,t+1}} - \left(\frac{\Pi_{t+1}r_{x,t+1}-1}{\Pi_{t+1}r_{x,t+1}}\right)\mu$$

The first-order condition for this problem is:

$$1 = E_t[sdf_{t+1}r_{x,t+1}] \tag{SA5}$$

The Euler equations for individual asset holdings can be obtained from (SA5) by setting $\lambda_{j,t}=1$ for $j \in \{k,n,i\}$ in the definition of $r_{x,t+1}$:

$$1 = E_t[sdf_{t+1}r_{k,t+1}] \qquad \text{for capital, } k \tag{SA6}$$

$$1 = E_t[sdf_{t+1}r_{n,t+1}] \qquad \text{for nominal bonds, } b_n \tag{SA7}$$

$$1 = r_t E_t[sdf_{t+1}] \qquad \text{for indexed bonds, } b_i \tag{SA8}$$

These are the standard Euler equations in the absence of a cash-in-advance constraint.

Equilibrium inflation and consumption by the old

The reserve requirement $m_t = \mu$ implies that $M_t/P_t = \mu$, where M_t is the nominal money stock and P_t is the aggregate price level. Consequently, inflation is equal to the money supply growth rate (see Equation (13) of the main text):

$$\Pi_t = P_t / P_{t-1} = \frac{\mu M_t}{\mu M_{t-1}} = \frac{M_t}{M_{t-1}} = \theta_t = \Phi^N \prod_{j=1}^{N} \exp(\varepsilon_{j,t}) \tag{SA9}$$

Turning to consumption by the old, note that since the monetary transfer is equal in real terms to $m_{t+1} - r_{m,t+1}m_t$ and $m_t = \mu$, we can write Equation (3) of the main text as follows:

$$\begin{aligned} c_{t+1,o} &= r_{k,t+1}k_{t+1} + r_t b_{i,t+1} + r_{n,t+1}b_{n,t+1} + m_{t+1} \\ &= r_{k,t+1}k_{t+1} + r_t b_{i,t+1} + r_{n,t+1}b_{n,t+1} + \mu \\ &= r_{k,t+1}k_{t+1} + [v_t r_t + (1-v_t)r_{n,t+1}]b_{t+1} + \mu \end{aligned} \tag{SA10}$$

where $v_t \equiv b_{i,t+1}/b_{t+1}$ is the share of government debt, $b_{t+1} = b_{i,t+1} + b_{n,t+1}$, that is indexed.

The inflation risk premium

Since $r_{n,t+1} = R_t/\Pi_{t+1}$, taking a second-order approximation of (SA7) and (SA8) leads to

$$\hat{r}_t = -E_t[s\hat{d}f_{t+1}] - \frac{1}{2}\text{var}_t[s\hat{d}f_{t+1}] \tag{SA11}$$

$$\begin{aligned} \hat{R}_t &= -E_t[s\hat{d}f_{t+1} - \hat{\Pi}_{t+1}] - \frac{1}{2}\text{var}_t[s\hat{d}f_{t+1} - \hat{\Pi}_{t+1}] \\ &= \hat{r}_t + E_t[\hat{\Pi}_{t+1}] - \frac{1}{2}\text{var}_t[\hat{\Pi}_{t+1}] + \text{cov}_t[s\hat{d}f_{t+1}, \hat{\Pi}_{t+1}] \end{aligned} \tag{SA12}$$

The inflation risk premium is the covariance term. We can derive an analytical expression for this term as follows. First, denoting the inflation risk premium *IRP*, we have

$$IRP = \text{cov}_t[s\hat{d}f_{t+1}, \hat{\Pi}_{t+1}] = -\gamma\,\text{cov}_t[\hat{c}_{t+1,O}, \hat{\Pi}_{t+1}] \tag{SA13}$$

Second, log-linearizing (SA10) and noting that $r = r_n$ at the deterministic steady-state:[32]

$$\begin{aligned} \hat{c}_{t+1,O} &= \theta_k[\hat{A}_{t+1} + \alpha\hat{k}_{t+1}] + \theta_b[v\hat{r}_t + (1-v)(\hat{R}_t - \hat{\Pi}_{t+1}) + \hat{b}_{t+1}] + b[r - r_n]\hat{v}_t \\ &= \theta_k[\hat{A}_{t+1} + \alpha\hat{k}_{t+1}] + \theta_b[v\hat{r}_t + (1-v)(\hat{R}_t - \hat{\Pi}_{t+1}) + \hat{b}_{t+1}] \end{aligned} \tag{SA14}$$

where $\theta_b \equiv rb / c_o$ and $\theta_k \equiv \alpha k^\alpha / c_o = 1 - \theta_b - \mu / c_o$.

Finally, substituting (SA14) into (SA13) and using (SA9) we find that

$$\begin{aligned} IRP &= -\gamma\,\text{cov}_t[\theta_k[\hat{A}_{t+1} + \alpha\hat{k}_{t+1}] + \theta_b[v\hat{r}_t + (1-v)(\hat{R}_t - \hat{\Pi}_{t+1}) + \hat{b}_{t+1}], \hat{\Pi}_{t+1}] \\ &= \gamma(1-v)\theta_b\,\text{var}_t[\hat{\Pi}_{t+1}] \\ &= \gamma(1-v)\theta_b N\sigma_M^2 > 0 \end{aligned} \tag{SA15}$$

This expression matches the one reported in Equation (22) of the main text.

[32] As in the main text, the indexation share is approximated around $v_t = v$, where $0 \le v \le 1$; see Equation (18) and Footnote 18. The resulting indexation share is constant at $v_t = v$ as per the discussion in the main text.