Imperfect Credibility and Robust Monetary Policy*

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Abstract

This paper studies the behavior of a central bank that seeks to conduct policy optimally while having imperfect credibility and harboring doubts about its model. Taking the Smets-Wouters model as the central bank's approximating model, the paper's main findings are as follows. First, a central bank's credibility can have large consequences for how policy responds to shocks. Second, central banks that have low credibility can benefit from a desire for robustness because this desire motivates the central bank to follow through on policy announcements that would otherwise not be time-consistent. Third, even relatively small departures from perfect credibility can produce important declines in policy performance. Finally, as a technical contribution, the paper develops a numerical procedure to solve the decision-problem facing an imperfectly credible policymaker that seeks robustness.

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1 Introduction

On August 9, 2011, against a background of heightened volatility in global financial markets, the Board of Governors of the Federal Reserve issued a monetary policy statement that read "*The Committee currently anticipates that economic conditions–including low rates of resource utilization and a subdued outlook for inflation over the medium run–are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013.*" This passage replaced the language in statements issued since December 16, 2008, which said "*The Committee continues to anticipate that economic conditions–including low rates of resource utilization and a subdued outlook for inflation over the medium run–are likely to warrant exceptionally low levels for the federal funds rate for an extended period.*" Similar passages can be found in more recent statements. Although the precise language has changed, each passage is notable for presenting households, firms, and investors with forward-guidance about monetary policy, guidance provided in an effort to leverage credibility in order to stimulate current economic activity. The passages are also notable in that the forward guidance is conditioned on a forecast for inflation and resource utilization, or slack. As a consequence, the effectiveness of the forward-guidance hinges on the Federal Reserve's credibility and on the potential for the forecasting model to be misspecified.

We consider the decision problem facing an imperfectly-credible central bank that seeks robustness to model uncertainty and explore the following questions. How important is credibility for monetary policy and macroeconomic outcomes? Does a central bank's desire for robustness help or hinder policymaking? How do imperfect credibility and robustness affect the forward-guidance that central bank's provide? The answers to these questions are important when central banks are relying increasingly on their credibility and on forward-guidance to gain leverage over current economic outcomes, all-the-while model uncertainty remains an ongoing concern.

To model credibility, we adopt the quasi-commitment approach developed by Roberds (1987), Schaumburg and Tambalotti (2007), and Debortoli and Nunes (2010). According to this literature a policymaker's credibility is associated with the probability that the promises it makes about future policy will be honored. Policymakers that have no credibility honor their promises with probability zero and conduct discretionary policy. Policymakers that have imperfect credibility honor their promises with probabilities between zero and one, with higher probabilities indicating higher credibility and a probability of one indicating commitment. Central banks desire higher levels of credibility because a lack of credibility leads to a (time-consistent) equilibrium characterized by a discretionary inflation bias and/or a discretionary stabilization bias. Under the former, the central bank, faced with the goals of keeping unemployment close to the natural rate and inflation close to target, succumbs to a short-run incentive to create surprise inflation with permanently higher inflation and no reduction in the unemployment rate the equilibrium outcome (Kydland and Prescott, 1977). Under the latter, the central bank, seeking to stabilize output and inflation efficiently in response to supply shocks, has an incentive to promise future policy interventions that mitigate the size of today's policy intervention, without having an incentive to subsequently deliver on those promises (Svensson, 1997; Clarida, Galí, and Gertler, 1999). The inefficiencies associated with both biases are overcome when credibility is perfect.

In addition to imperfect credibility, the central bank that we study is concerned about model misspecification. To model the central bank's concern for model misspecification we adopt the robust control approach advanced by Hansen and Sargent (2008). According to the robust control literature, a policymaker that desires robustness against model misspecification will formulate policy in the context of a potentially distorted, or misspecified, approximating model so as to guard against the worst permissible misspecification. Through this mechanism the policymaker is able to conduct model-based policy while also expressing distrust in its model.

After developing the decision problem confronting an imperfectly credibility policymaker that seeks robustness to model uncertainty and presenting its solution, we use the Smets and Wouters (2007) model to examine the effects that imperfect credibility and robustness have on optimal policymaking. We employ the Smets and Wouters (2007) model for our analysis because it is widely understood, it forms the basis for many other models, and it is thought to fit U.S. data well; in these respects it can usefully be viewed as the central bank's approximating model. Moreover, the Smets-Wouters model contains a broad array of shocks whose presence provides ample cover for model misspecification and it is forward-looking allowing policy announcements and central bank credibility to potentially play important roles. A further advantage to using the Smets-Wouters model is that our qualitative findings are likely to generalize to the many related models.

The main lessons that emerge are the following. First, a central bank's credibility gives it a powerful lever for managing private-sector expectations and for stabilizing the economy.

Second, when a central bank has low credibility the economy can benefit from the central bank's desire for robustness. Put differently, the central bank's desire for robustness can act somewhat as a substitute for credibility when credibility is low. This result emerges because a robust central bank is directed to respond aggressively to stabilize inflation following shocks, pursuing a policy that would ordinarily be infeasible for a central bank that lacks credibility. Third, even relatively small departures from perfect credibility produce big declines in policy performance, giving rise to a form of discretionary stabilization bias. The over-riding lesson that emerges from this analysis is that credibility is extremely valuable for central banks, both when the model is known to be correctly specified and when it is suspected that it is not.

In addition to the work of Schaumburg and Tambalotti (2007), Debortoli and Nunes (2010), and Hansen and Sargent (2008), this paper is related to Bodenstein, Hebden, and Nunes (2010) and Kasa (2002). However, where Bodenstein, Hebden, and Nunes (2010) focus on the interaction between imperfect credibility and the zero-bound on nominal interest rates, we focus on the interaction between imperfect credibility and model uncertainty. Nonetheless, our results are consistent with theirs in-so-much as we too find that policymakers tend to make more extreme policy announcements as their credibility declines. Like ourselves, Kasa (2002) uses robust control to analyze the effects of model uncertainty on policy design in a model where private agents are forward-looking. But unlike ourselves, Kasa (2002) analyzes use frequency domain methods to analyze the robustness of a simple stylized New Keynesian model and looks at commitment from a timeless perspective (Woodford, 1999).

The remainder of this paper is structured as follows. Section 2 describes the decision problem facing a central bank that seeks to guard against model misspecification while endowed with imperfect credibility. Section 3 establishes the connection between robust control and risk-sensitive preferences for this class of quasi-commitment decision problems. Section 4 summarizes and analyzes the Smets-Wouters model that serves as our laboratory for analysis. Section 5 concludes.

2 Robustness and imperfect credibility

In this section we describe a linear-quadratic planning problem and characterize its solution. This planning problem involves constraints that contain non-predetermined variables and is related to the commitment problems that are solved routinely in the monetary policy literature, while differing in two important respects. First, the decisionmaking environment is one in which the policymaker has imperfect credibility. Imperfect credibility is modeled according to the quasi-commitment literature which allows the policymaker to stochastically default, reoptimizing its plan at stochastic intervals. In this aspect, the analysis builds on work by Roberds (1987), Schaumburg and Tambalotti (2007), and Debortoli and Nunes (2010). Second, the decisionmaking environment is one in which the policymaker has doubts about its model and seeks a policy that is robust in the sense of Hansen and Sargent (2008). In this aspect, the analysis is related to work by Giordani and Söderlind (2004), Hansen and Sargent (2008, chapter 16), and Dennis (2008, 2010).

The two key parameters in the decision problem that we formulate are $\alpha \in [0,1]$, which governs the policymaker's credibility, and $\theta \in [\underline{\theta}, \infty)$, which governs the policymaker's distrust in its model. Importantly, many standard decisionmaking problems emerge as special cases of this decision problem. Specifically, for different limiting values of α and θ the decision problem simplifies to nonrobust commitment ($\theta \uparrow \infty$, $\alpha \uparrow 1$), nonrobust discretion ($\theta \uparrow \infty$, $\alpha \downarrow 0$), robust commitment ($\alpha \uparrow 1 | \theta \in [\underline{\theta}, \infty)$), robust discretion ($\alpha \downarrow 0 | \theta \in [\underline{\theta}, \infty)$), and quasi-commitment ($\theta \uparrow \infty | \alpha \in [0,1]$).

2.1 The approximating model

The economy consists of households, firms, and a policymaker, which in our application is a central bank. All agents are assumed to share an approximating model that they believe comes closest to describing the process governing economic outcomes. According to this approximating model, an $n \times 1$ vector of endogenous variables, $\mathbf{z}_t$, consisting of n_1 predetermined variables, $\mathbf{x}_t$, and n_2 ($n_2 = n - n_1$) nonpredetermined variables, $\mathbf{y}_t$, evolves over time according to

$$\mathbf{x}_{t+1} = \mathbf{A}_{11}\mathbf{x}_t + \mathbf{A}_{12}\mathbf{y}_t + \mathbf{B}_1\mathbf{u}_t + \mathbf{C}_1\boldsymbol{\varepsilon}_{\mathbf{x}t+1}, \tag{1}$$

$$\mathbf{A}_0 \mathrm{E}_t \mathbf{y}_{t+1} = \mathbf{A}_{21}\mathbf{x}_t + \mathbf{A}_{22}\mathbf{y}_t + \mathbf{B}_2\mathbf{u}_t, \tag{2}$$

where $\mathbf{u}_t$ is a $p \times 1$ vector of policy control variables and $\boldsymbol{\varepsilon}_{\mathbf{x}t} \sim i.i.d.\,[\mathbf{0}, \mathbf{I}]$ is an $n_\varepsilon \times 1$ ($n_\varepsilon \leq n_1$) vector of white-noise innovations. The matrices $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21}$, $\mathbf{A}_{22}$, $\mathbf{B}_1$, and $\mathbf{B}_2$ are conformable with $\mathbf{x}_t$, $\mathbf{y}_t$, and $\mathbf{u}_t$ as necessary while the matrix $\mathbf{C}_1$ is constructed to ensure that $\boldsymbol{\varepsilon}_{\mathbf{x}t}$ has the identity matrix as its variance-covariance matrix. The operator E_t represents the private sector's mathematical expectation operator conditional upon period t information. Equation (2) accommodates a leading matrix $\mathbf{A}_0$ that need not have full rank.

Equations (1) and (2) are standard constraints in linear-quadratic decision problems in which private agents are forward-looking and policy is conducted under either commitment or discretion (Currie and Levine, 1993) or under timeless-perspective commitment (Woodford, 2010; Svensson, 2010). Of course, when policy is conducted under discretion equations (1) and (2) must be augmented with an equation of the form

$$\mathrm{E}_t \mathbf{y}_{t+1} = \mathbf{H}\mathrm{E}_t \mathbf{x}_{t+1}, \tag{3}$$

where $\mathbf{H}$ is determined in equilibrium, to account to the fact that private-sector expectations depend only on the state variables in a Markov-perfect (and hence time-consistent) equilibrium (Kydland and Prescott, 1977).

2.1.1 Introducing imperfect credibility

Building on Roberds (1987), Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010) analyze decision problems in rational expectations models that are characterized by what they call "quasi-commitment." Quasi-commitment provides a workable approach to modeling imperfect credibility because it allows a separation between what policymakers say they are going to do and what they end up doing.

Quasi-commitment views commitment and discretion as opposite ends of a unit-continuum of decision-problems. Each decision-problem on the unit-continuum is indexed by $\alpha \in [0, 1]$, where α denotes the mean of the random variable η_t, which obeys a Bernoulli distribution. The underlying environment can be interpreted several ways. One interpretation is that the environment is one in which the policymaker makes announcements about future policy with all agents (including the policymaker) making decisions knowing that the announced policy will only be implemented with probability α. An alternative interpretation is that the environment is one in which policymakers can credibly commit to a state-contingent plan, or policy, for the duration of their tenure, but where each policymaker's tenure is uncertain, governed by the outcome of a sequence of *i.i.d.* draws of the random variable η_t. Accordingly, if $\eta_t = 1$, then the incumbent-policymaker's tenure continues in period t, whereas if $\eta_t = 0$, then the incumbent-policymaker's tenure ends at the beginning of period t. In the event that the incumbent-policymaker's tenure ends, that policymaker is replaced by another with identical preferences, but that is not beholden to honor the policies announced by any of its predecessors. Under either interpretation, $\alpha = 1$ corresponds to commitment, $\alpha = 0$

corresponds to discretion, and $\alpha \in (0,1)$ corresponds to a form of limited commitment or imperfect credibility.

At the start of every period a draw for η_t is received and is observed by all agents. In forming their period-t expectations of $\mathbf{y}_{t+1}$, therefore, private agents take into account uncertainty about the shocks hitting the economy and uncertainty about whether the incumbent or a new policymaker will be conducting policy in period $t+1$. Assuming that the Bernoulli distribution that governs η_t is independent of the probability density that governs the innovations, $\boldsymbol{\varepsilon}_{\mathbf{x}t}$, equation (2) can be written as

$$\mathbf{A}_0 \mathrm{E}_t \mathbf{y}_{t+1} = \alpha \mathbf{A}_0 \mathrm{E}_t \left[\mathbf{y}_{t+1} | \left(\eta_{t+1} = 1 \right) \right] + (1-\alpha) \mathbf{A}_0 \mathrm{E}_t \left[\mathbf{y}_{t+1} | \left(\eta_{t+1} = 0 \right) \right], \tag{4}$$

where the expectation $\mathrm{E}_t \left(\mathbf{y}_{t+1} | \eta_{t+1} = 0 \right)$ is governed by an expression that takes the form of equation (3).

2.1.2 Introducing model uncertainty

Following Hansen and Sargent (2008), the policymaker does not fully trust the approximating model, fearing that it may be misspecified. Although it fears that its approximating model is misspecified, the policymaker believes that private agents know the correct model. Thus the robust decision problem formulated below follows Hansen and Sargent's (2012) first type of ambiguity.

To accommodate the policymaker's concerns, distortions, or specification errors, $\mathbf{v}_{t+1}$, are introduced, disguised by the innovations, $\boldsymbol{\varepsilon}_{\mathbf{x}t+1}$. A consequence of the specification errors is that equation (1) in the approximating model becomes

$$\mathbf{x}_{t+1} = \mathbf{A}_{11} \mathbf{x}_t + \mathbf{A}_{12} \mathbf{y}_t + \mathbf{B}_1 \mathbf{u}_t + \mathbf{C}_1 \left(\mathbf{v}_{t+1} + \boldsymbol{\varepsilon}_{\mathbf{x}t+1} \right), \tag{5}$$

in the "distorted" model and that equation (2) in the approximating model becomes

$$\begin{aligned} \alpha \mathbf{A}_0 \mathbf{y}_{t+1} | \left(\eta_{t+1} = 1 \right) &= \left[\mathbf{A}_{21} - (1-\alpha) \mathbf{A}_0 \mathbf{H} \mathbf{A}_{11} \right] \mathbf{x}_t + \left[\mathbf{A}_{22} - (1-\alpha) \mathbf{A}_0 \mathbf{H} \mathbf{A}_{12} \right] \mathbf{y}_t \\ &\quad + \left[\mathbf{B}_2 - (1-\alpha) \mathbf{A}_0 \mathbf{H} \mathbf{B}_1 \right] \mathbf{u}_t + \alpha \mathbf{A}_0 \mathbf{C}_2 \left(\mathbf{v}_{t+1} + \boldsymbol{\varepsilon}_{\mathbf{x}t+1} \right), \end{aligned} \tag{6}$$

in the distorted model, where $\mathbf{H}$ and $\mathbf{C}_2$ have yet to be determined. In equation (6), $\mathbf{H}$ characterizes the relationship between the non-predetermined variables, $\mathbf{y}_t$, and the predetermined variables, $\mathbf{x}_t$, in the event that a reoptimization occurs ($\eta_t = 0$) while $\mathbf{C}_2$ summarizes how errors in forecasting the non-predetermined variables (i.e. $\mathbf{y}_{t+1} | \left(\eta_{t+1} = 1 \right) - \mathrm{E}_t \left[\mathbf{y}_{t+1} | \left(\eta_{t+1} = 1 \right) \right]$)

are related to the innovations, $\boldsymbol{\varepsilon}_{\mathbf{x}t+1}$. More compactly, and in obvious notation, equation (6) can be written as

$$\mathbf{D}_0\mathbf{y}_{t+1}|\left(\eta_{t+1}=1\right)=\mathbf{D}_1\mathbf{x}_t+\mathbf{D}_2\mathbf{y}_t+\mathbf{D}_3\mathbf{u}_t+\mathbf{D}_4\mathbf{v}_{t+1}+\mathbf{D}_4\boldsymbol{\varepsilon}_{\mathbf{x}t+1}. \tag{7}$$

The sequence of specification errors, $\{\mathbf{v}_{s+1}\}_{s=t}^{\infty}$ is constrained to satisfy the boundedness condition

$$\beta\mathrm{E}\sum_{s=t}^{\infty}\beta^{(s-t)}\mathbf{v}_{s+1}^{'}\mathbf{v}_{s+1}\leq\omega, \tag{8}$$

where $\omega\in[0,\overline{\omega}]$. It is the satisfaction of this boundedness condition that defines the sense in which the approximating model, summarized by equations (1)—(2), is a good one. When $\omega=0$, the policymaker trusts the approximating model and conducts policy as if the approximating model is correct. As ω increases, however, the policymaker increasingly suspects that the approximating model is misspecified. For $\omega>\overline{\omega}$, the policymaker's doubts about the approximating model are such that it no longer views the approximating model to be a good representation of the data-generating process.

2.2 The robust decision problem with imperfect credibility

The policymaker's objective function is given by the loss function

$$\mathrm{E}\sum_{s=t}^{\infty}\beta^{(s-t)}L\left(\mathbf{x}_s,\mathbf{y}_s,\mathbf{u}_s\right), \tag{9}$$

where $\beta\in(0,1)$ is the discount factor and $L\left(\mathbf{x}_s,\mathbf{y}_s,\mathbf{u}_s\right)$ is quadratic and convex to the origin.

As noted earlier, in the event that $\eta_t=1$, the incumbent policymaker's tenure continues. However, in the event that $\eta_t=0$, the period-t decision problem for the newly-appointed policymaker is to choose $\{\mathbf{u}_s\}_{s=t}^{\infty}$ to minimize and $\{\mathbf{v}_{s+1}\}_{s=t}^{\infty}$ to maximize equation (9) subject to equations (5), (7), and (8), and $\mathbf{x}_t$ known. According to this decision problem, to guard against the specification errors that it fears, the robust policymaker formulates policy subject to the distorted model with the mind-set that the specification errors will be as damaging as possible, a view operationalized via the metaphor that $\{\mathbf{v}_{s+1}\}_{s=t}^{\infty}$ is chosen by a fictitious evil agent whose objectives are diametrically opposed to those of the policymaker. Following Hansen and Sargent (2008), this constraint problem can be replaced with an equivalent multiplier problem, in which

$$\mathrm{E}\sum_{s=t}^{\infty}\beta^{(s-t)}\left[L\left(\mathbf{x}_s,\mathbf{y}_s,\mathbf{u}_s\right)-\beta\theta\mathbf{v}_{s+1}^{'}\mathbf{v}_{s+1}\right], \tag{10}$$

$\theta \in [\underline{\theta}, \infty)$, is maximized with respect to $\{\mathbf{v}_{s+1}\}_{s=t}^{\infty}$ and minimized with respect to $\{\mathbf{u}_s\}_{s=t}^{\infty}$, subject to equations (5) and (7), and $\mathbf{x}_t$ known. The multiplier, or robustness parameter, θ, represents the shadow price of a marginal relaxation in the boundedness condition, equation (8). Larger values for θ, which correspond to smaller values of ω, signify greater confidence in the adequacy of the approximating model. Of course, in the limit as $\theta \uparrow \infty$, the nonrobust decision problem is restored.

From the Lagrange function

$$\Lambda_t = \mathrm{E}\sum_{s=t}^{\infty} \beta^{(s-t)} \left[\begin{array}{c} L(\mathbf{x}_s, \mathbf{y}_s, \mathbf{u}_s) - \beta\theta \mathbf{v}_{s+1}^{'} \mathbf{v}_{s+1} \\ -2\boldsymbol{\Xi}_s^{'} \left(\mathbf{D}_1\mathbf{x}_s + \mathbf{D}_2\mathbf{y}_s + \mathbf{D}_3\mathbf{u}_s + \mathbf{D}_4\mathbf{v}_{s+1} + \mathbf{D}_5\boldsymbol{\varepsilon}_{\mathbf{x}s+1} - \mathbf{D}_0\mathbf{y}_{s+1} | \left(\eta_{s+1} = 1\right)\right) \\ -2\lambda_s^{'} \left(\mathbf{A}_{11}\mathbf{x}_s + \mathbf{A}_{12}\mathbf{y}_s + \mathbf{B}_1\mathbf{u}_s + \mathbf{C}_1 \left(\mathbf{v}_{s+1} + \boldsymbol{\varepsilon}_{\mathbf{x}s+1} \right) - \mathbf{x}_{s+1} \right) \end{array} \right], \tag{11}$$

we construct the "dual" loss function

$$\begin{aligned} \widetilde{L}(\mathbf{x}_s, \boldsymbol{\Xi}_{s-1}, \mathbf{y}_s, \mathbf{u}_s, \boldsymbol{\gamma}_s, \mathbf{v}_{s+1}) \quad = \quad & L(\mathbf{x}_s, \mathbf{y}_s, \mathbf{u}_s) - \beta\theta \mathbf{v}_{s+1}^{'} \mathbf{v}_{s+1} \\ & -2\boldsymbol{\gamma}_s^{'} \left(\mathbf{D}_1\mathbf{x}_s + \mathbf{D}_2\mathbf{y}_s + \mathbf{D}_3\mathbf{u}_s + \mathbf{D}_4\mathbf{v}_{s+1} + \mathbf{D}_5\boldsymbol{\varepsilon}_{\mathbf{x}s+1} \right) \\ & +2\boldsymbol{\Xi}_{s-1}^{'} \mathbf{D}_0 \mathbf{y}_s, \end{aligned} \tag{12}$$

where $\boldsymbol{\gamma}_s = \boldsymbol{\Xi}_s$, allowing equation (11) to be expressed as

$$\Lambda_t = \mathrm{E}\sum_{s=t}^{\infty} \beta^{(s-t)} \left[\begin{array}{c} \widetilde{L}(\mathbf{x}_s, \boldsymbol{\Xi}_{s-1}, \mathbf{y}_s, \mathbf{u}_s, \boldsymbol{\gamma}_s, \mathbf{v}_{s+1}) \\ -2\lambda_s^{'} \left(\mathbf{A}_{11}\mathbf{x}_s + \mathbf{A}_{12}\mathbf{y}_s + \mathbf{B}_1\mathbf{u}_s + \mathbf{C}_1 \left(\mathbf{v}_{s+1} + \boldsymbol{\varepsilon}_{\mathbf{x}s+1} \right) - \mathbf{x}_{s+1} \right) \end{array} \right]. \tag{13}$$

Now defining $\widetilde{\mathbf{X}}_t = \left[\begin{array}{cc} \mathbf{x}_t^{'} & \boldsymbol{\Xi}_{t-1}^{'} \end{array} \right]^{'}$ and $\widetilde{\mathbf{u}}_t = \left[\begin{array}{ccc} \mathbf{y}_t^{'} & \mathbf{u}_t^{'} & \boldsymbol{\gamma}_t^{'} \end{array} \right]^{'}$ and employing Marcet and Marimon's recursive saddle-point theorem (Marcet and Marimon, 2009), the robust decision problem for the newly-appointed policymaker can be expressed in terms of the Bellman equation

$$\widetilde{\mathbf{X}}_t^{\prime} \mathbf{V} \widetilde{\mathbf{X}}_t + d = \max_{(\boldsymbol{\gamma}_t)} \min_{(\mathbf{y}_t, \mathbf{u}_t)} \max_{(\mathbf{v}_{t+1})} \left[\widetilde{L}\left(\widetilde{\mathbf{X}}_t, \widetilde{\mathbf{u}}_t, \mathbf{v}_{t+1} \right) + \beta \mathrm{E}_t \left(\widetilde{\mathbf{X}}_{t+1}^{\prime} \widetilde{\mathbf{V}} \widetilde{\mathbf{X}}_{t+1} + d \right) \right], \tag{14}$$

in which

$$\widetilde{\mathbf{V}} = \alpha \mathbf{V} + (1-\alpha) \mathbf{S}^{\prime} \mathbf{S}^{\prime -1} \mathbf{V} \mathbf{S}^{-1} \mathbf{S}, \tag{15}$$

with $\mathbf{S} = \left[\begin{array}{cc} \mathbf{I} & \mathbf{0} \end{array} \right]$ (and $\mathbf{S}^{-1}$ representing the generalized left inverse of $\mathbf{S}$), subject to

$$\widetilde{\mathbf{X}}_{t+1} = \widetilde{\mathbf{A}} \widetilde{\mathbf{X}}_t + \widetilde{\mathbf{B}} \widetilde{\mathbf{u}}_t + \widetilde{\mathbf{C}} \mathbf{v}_{t+1} + \widetilde{\mathbf{C}} \boldsymbol{\varepsilon}_{\mathbf{x}t+1}. \tag{16}$$

In equation (16), the system matrices are given by $\widetilde{\mathbf{A}} = \left[\begin{array}{cc} \mathbf{A}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array} \right]$, $\widetilde{\mathbf{B}} = \left[\begin{array}{ccc} \mathbf{A}_{12} & \mathbf{B}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{array} \right]$, and $\widetilde{\mathbf{C}} = \left[\begin{array}{c} \mathbf{C}_1 \\ \mathbf{0} \end{array} \right]$.

The decision problem described by equations (14)—(16) is essentially an optimal linear regulator problem that can be solved using standard methods. From its solution it is straightforward to recover updated terms for $\mathbf{H}$ and $\mathbf{C}_2$. Beginning with conjectured values for $\mathbf{H}$ and $\mathbf{C}_2$, iterating to convergence delivers the worst-case decision rules and the worst-case equilibrium law-of-motion.[1] With the worst-case equilibrium in hand, it is straightforward to recover the approximating equilibrium. In the approximating equilibrium, although the policymaker employs its robust decision rule, the approximating model is taken to be correctly specified. The approximating equilibrium gives us an equilibrium law-of-motion

$$\widetilde{\mathbf{X}}_{t+1} = \left(\widetilde{\mathbf{A}} + \widetilde{\mathbf{B}}\mathbf{F}\right)\widetilde{\mathbf{X}}_t + \widetilde{\mathbf{C}}\boldsymbol{\varepsilon}_{\mathbf{x}t+1}, \tag{17}$$

and a collection of decision rules

$$\widetilde{\mathbf{u}}_t = \mathbf{F}\widetilde{\mathbf{X}}_t. \tag{18}$$

Notice that the worst-case equilibrium and the approximating equilibrium are both expressed in terms of a state vector that includes the multipliers, $\boldsymbol{\Xi}_{t-1}$. Accordingly, in the event that $\eta_t = 0$ the approximating equilibrium is governed by equations (17)—(18), but with $\boldsymbol{\Xi}_{t-1} = \mathbf{0}$.

3 A risk-sensitive formulation

For linear-quadratic infinite-horizon discounted stochastic models in which the constraints do not contain nonpredetermined variables, Hansen and Sargent (2008, chapter 2) show that the decision rule that solves the robust control problem also solves an alternative infinite-horizon discounted stochastic decision problem in which the policymaker does not fear model misspecification, but instead has risk-sensitive preferences (Whittle, 1990). We extend that result to models whose constraints do contain nonpredetermined variables, focusing here on the case where policy is conducted with discretion. The general case where the constraints contain nonpredetermined variables and policy is conducted with quasi-commitment is treated in Appendix A.

The connection between the robust control formulation and the risk-sensitive preferences formulation is useful for several reasons. It links the robust control problem to ambiguity/uncertainty aversion and offers a more general interpretation of the decision problem as a

[1] This solution procedure has worked well when applied to the Smets-Wouters model and to other models, converging rapidly and without difficulty.

consequence. Specifically, the policymaker's doubts about the model lead to behavior that can equivalently be generated by additional sensitivity to risk (Whittle, 1990), Epstein-Zin-preferences (Epstein and Zin, 1989), or ambiguity aversion (Gilboa and Schmeidler, 1989).

With policy conducted under pure discretion ($\alpha = 0$), it follows from equation (6) that the aggregate reaction function for the nonpredetermined variables can be written as

$$\mathbf{y}_t = \mathbf{J}\mathbf{x}_t + \mathbf{K}\mathbf{u}_t, \tag{19}$$

where

$$\begin{aligned}\mathbf{J} &= \left[\mathbf{A}_{22} - \mathbf{A}_0\mathbf{H}\mathbf{A}_{12}\right]^{-1}\left[\mathbf{A}_0\mathbf{H}\mathbf{A}_{11} - \mathbf{A}_{21}\right], \\ \mathbf{K} &= \left[\mathbf{A}_{22} - \mathbf{A}_0\mathbf{H}\mathbf{A}_{12}\right]^{-1}\left[\mathbf{A}_0\mathbf{H}\mathbf{B}_1 - \mathbf{B}_2\right].\end{aligned}$$

Equation (19) applies under both the approximating model and the distorted model. Given equation (19), the law-of-motion for the predetermined variables in the approximating model and in the distorted model are

$$\mathbf{x}_{t+1} = \left[\mathbf{A}_{11} + \mathbf{A}_{12}\mathbf{J}\right]\mathbf{x}_t + \left[\mathbf{B}_1 + \mathbf{A}_{12}\mathbf{K}\right]\mathbf{u}_t + \mathbf{C}_1\boldsymbol{\varepsilon}_{\mathbf{x}t+1}, \tag{20}$$

and

$$\mathbf{x}^*_{t+1} = \left[\mathbf{A}_{11} + \mathbf{A}_{12}\mathbf{J}\right]\mathbf{x}_t + \left[\mathbf{B}_1 + \mathbf{A}_{12}\mathbf{K}\right]\mathbf{u}_t + \mathbf{C}_1\left(\mathbf{v}_{t+1} + \boldsymbol{\varepsilon}_{\mathbf{x}t+1}\right), \tag{21}$$

respectively.

The risk-sensitive formulation employs equation (20) (because the policymaker trusts the model), and leads to the Bellman-equation

$$\mathbf{x}_t'\mathbf{V}\mathbf{x}_t + d = \min_{u_t}\left[L\left(\mathbf{x}_t, \mathbf{J}\mathbf{x}_t + \mathbf{K}\mathbf{u}_t, \mathbf{u}_t\right) - \frac{1}{\sigma}\ln\left(\mathrm{E}_t\left(\exp\left(\sigma\beta\left(\mathbf{x}_{t+1}'\mathbf{V}\mathbf{x}_{t+1} + d\right)\right)\right)\right)\right], \tag{22}$$

where the risk-sensitivity parameter satisfies $\sigma \leq 0$ and $\mathbf{V}$ is positive semi-definite. Employing a result from Jacobson (1973), equation (22) is equivalent to

$$\mathbf{x}_t'\mathbf{V}\mathbf{x}_t + d = \min_{u_t}\left[L\left(\mathbf{x}_t, \mathbf{J}\mathbf{x}_t + \mathbf{K}\mathbf{u}_t, \mathbf{u}_t\right) + \beta\mathrm{E}_t\left(\mathbf{x}_{t+1}'D\left(\mathbf{V}\right)\mathbf{x}_{t+1} + \widehat{d}\right)\right], \tag{23}$$

where

$$D\left(\mathbf{V}\right) = \mathbf{V} - \sigma\mathbf{V}\mathbf{C}_1\left(\mathbf{I} + \sigma\mathbf{C}_1'\mathbf{V}\mathbf{C}_1\right)^{-1}\mathbf{C}_1'\mathbf{V}.$$

In contrast, the robust-control formulation employs equation (21) (because the policymaker distrusts the model), and leads to the Bellman-equation

$$\mathbf{x}_t'\mathbf{V}\mathbf{x}_t + d = \min_{u_t}\max_{v_{t+1}}\left[L\left(\mathbf{x}_t, \mathbf{J}\mathbf{x}_t + \mathbf{K}\mathbf{u}_t, \mathbf{u}_t\right) - \beta\theta\mathbf{v}_{t+1}'\mathbf{v}_{t+1} + \beta\mathrm{E}_t\left(\mathbf{x}^{*\prime}_{t+1}\mathbf{V}\mathbf{x}^*_{t+1} + d\right)\right]. \tag{24}$$

Performing the inner-maximization gives

$$\mathbf{v}_{t+1} = \frac{1}{\theta}\mathrm{E}_t\left[\left(\mathbf{I} - \frac{1}{\theta}\mathbf{C}_1^{'}\mathbf{V}\mathbf{C}_1\right)^{-1}\mathbf{C}_1^{'}\mathbf{V}\mathbf{x}_{t+1}\right]. \tag{25}$$

Substituting equations (25) and (21) into equation (24) results in

$$\mathbf{x}_t^{'}\mathbf{V}\mathbf{x}_t + d = \min_{u_t}\left[\begin{array}{c} L\left(\mathbf{x}_t, \mathbf{J}\mathbf{x}_t + \mathbf{K}\mathbf{u}_t, \mathbf{u}_t\right) \\ +\beta\mathrm{E}_t\left(\mathbf{x}_{t+1}^{'}\left[\mathbf{V} + \frac{1}{\theta}\mathbf{V}\mathbf{C}_1\left(\mathbf{I} - \frac{1}{\theta}\mathbf{C}_1^{'}\mathbf{V}\mathbf{C}_1\right)^{-1}\mathbf{C}_1^{'}\mathbf{V}\right]\mathbf{x}_{t+1} + \overline{d}\right)\end{array}\right], \tag{26}$$

which, aside from the difference between $\widehat{d}$ and $\overline{d}$ (which does not affect the decision rules), is equivalent to equation (23) with $\theta = -\sigma^{-1}$. Importantly, it is the approximating model, equation (20), that constrains equation (26).

The treatment above parallels Hansen and Sargent's (2008, chapter 2) treatment of the optimal linear regulator problem. The two problems are related because the aggregate reaction function (equation 19) allows the nonpredetermined variables to be eliminated from the system leading to a recursive problem in which the state variables are given by $\mathbf{x}_t$. The connection between equation (26) and Epstein-Zin-preferences comes from the fact that risk-sensitive preferences are a special case of Epstein-Zin preferences. Finally, the connection between equation (26) and ambiguity/uncertainty aversion follows from Hansen and Sargent (2007).

4 The model in summary

To examine robust policymaking with imperfect credibility we use as our approximating model the Smets and Wouters (2007) model for the U.S. We use this model for several reasons. First, the model has been found to provide a reasonably good description of U.S. economic outcomes. Second, the model forms the basis for many related models and its widespread usage, together with its empirical support, make it a sensible choice. Third, the model's structure accommodates many shocks, which from the robust control perspective, represent sources of potential misspecification. Fourth, private agents are forward-looking, allowing central-bank credibility to influence private-sector decisionmaking.

Because the Smets and Wouters (2007) model is widely known, we describe only its main characteristics here and refer interested readers to the original text. The economy is populated by three types of agents: households, firms, and a central bank. Households own the capital stock and the equity in firms and receive income from dividends and from renting capital and supplying labor to firms. Households use their income to purchase goods that they allocate between consumption and investment in order to maximize expected lifetime utility. Goods allocated to investment augment the capital stock, subject to a resource cost associated with changing the investment-flow. Households gain utility from consumption (subject to an external consumption habit) and from leisure, and they are monopolistically competitive suppliers of their labor, setting their wage subject to a Calvo-style wage rigidity. Those households that are unable to change their wage in a given period are assumed to index their wage to lagged aggregate inflation. On the production side, firms are monopolistically competitive; they rent capital and hire labor and produce according to a constant-returns Cobb-Douglas production function. Firms choose how much capital and labor to employ and set prices in order to maximize the expected present discounted value of the firm, subject to a Calvo (1983) price rigidity and price indexation. Profits are returned to households in the form of a lump-sum dividend. Finally, the goods that firms produce are combined according to a Kimball (1995) technology to produce a final good that is sold to households in a perfectly competitive market.

Although Smets and Wouters (2007) characterize monetary policy in terms of an estimated Taylor-type rule, our focus is on optimal policymaking. Accordingly, we take the "primal" approach and replace their estimated policy rule with one chosen in order to minimize the following loss function

$$L(\mathbf{x}_t, \mathbf{y}_t, \mathbf{u}_t) = \pi_t^2 + \mu \left(y_t - y_t^f \right)^2, \tag{27}$$

where π_t denotes annualized quarterly inflation, y_t denotes output, y_t^f denotes flex-price output, and $y_t - y_t^f$ denotes the output gap. The parameter $\mu \in [0, \infty)$ governs the weight assigned to stabilizing the output gap relative to stabilizing inflation. The model is log-linearized about a zero-inflation balanced growth path and is subject to six shocks, including those to the aggregate production technology, the investment-specific production technology, and to the price and wage markups. These six shocks obscure potential specification errors. We parameterize the model using the coefficient estimates provided by the posterior mean (Smets and Wouters, 2007, Tables 1A and 1B) and choose $\mu = 0.25$ as a benchmark value.

5 How large is the discretionary stabilization bias?

Before looking at the effects that imperfect credibility have on the model, we first quantify the magnitude of the stabilization bias for different values of μ. Following Dennis and Söderström

(2006), we quantify the stabilization bias by calculating the percent gain in loss associated with having a commitment technology, which is given by

$$\Omega = 100 \times \left[1 - \frac{V^c}{V^d}\right], \tag{28}$$

where V^c and V^d represent losses under commitment and discretion, respectively, and by calculating the inflation equivalent (Jensen, 2002; Dennis and Söderström, 2006), which is given by

$$\widehat{\pi} = \sqrt{V^d - V^c}. \tag{29}$$

The interpretation of Ω, the percent gain in loss associated with commitment is straightforward. However, as a measure of stabilization bias it suffers from the problem that where the losses under commitment and discretion are both small, large percentage gains can be attributed to commitment although the absolute difference in losses is small. The inflation equivalent measures the amount by which the central bank could permanently miss its inflation target under commitment and still be no worse than discretion.

Table 1: Stabilization Bias in Smets and Wouters (2007)

μ	V^c	V^d	% Gain from commitment	Inflation equivalent
0.25	3.105	12.277	74.713	3.029
0.50	4.441	13.175	66.294	2.955
1.00	6.077	13.676	55.564	2.757
2.00	7.983	13.941	42.736	2.441
4.00	10.180	14.078	27.691	1.974

Table 1 displays the losses under commitment and discretion, the percent gain from commitment, and the inflation equivalent for a range of values for μ. Importantly, for all values of μ considered both the percent gain from commitment and the inflation equivalent are large, signalling that the absence of commitment, by leading to a discretionary stabilization bias, has large effects in the model. With μ equal to 0.25, the central bank could miss its inflation target by a full three percentage points under commitment and this outcome still be preferred to a discretionary equilibrium in which the inflation target is hit on average. The finding that stabilization bias is large in this model is consistent with Dennis and Söderström (2006), who showed that the discretionary stabilization bias tends to be larger in models that lack transmission lags and in which expectations are formed using period-t information, both characteristics of the Smets-Wouters model.

To relate this analysis back to Schaumburg and Tambalotti (2007), Figure 1 displays the relationship between the average regime duration, $(1-\alpha)^{-1}$ and the gain to credibility that remains, where the latter is measured by

$$\Theta(\alpha) = \frac{V^{qc}(\alpha) - V^{c}}{V^{d} - V^{c}}.$$

When $\alpha = 0$, $V^{qc}(0) = V^{d}$ and $\Theta(0) = 1$. Then, as α increases and approaches 1, expected loss with quasi-commitment, $V^{qc}(\alpha)$, approaches expected loss with commitment, V^{c}, and $\Theta(\alpha)$ approaches 0. For any given level of α, $\Theta(\alpha)$ reports how much of the gap between discretion and commitment remains to be closed through higher credibility.

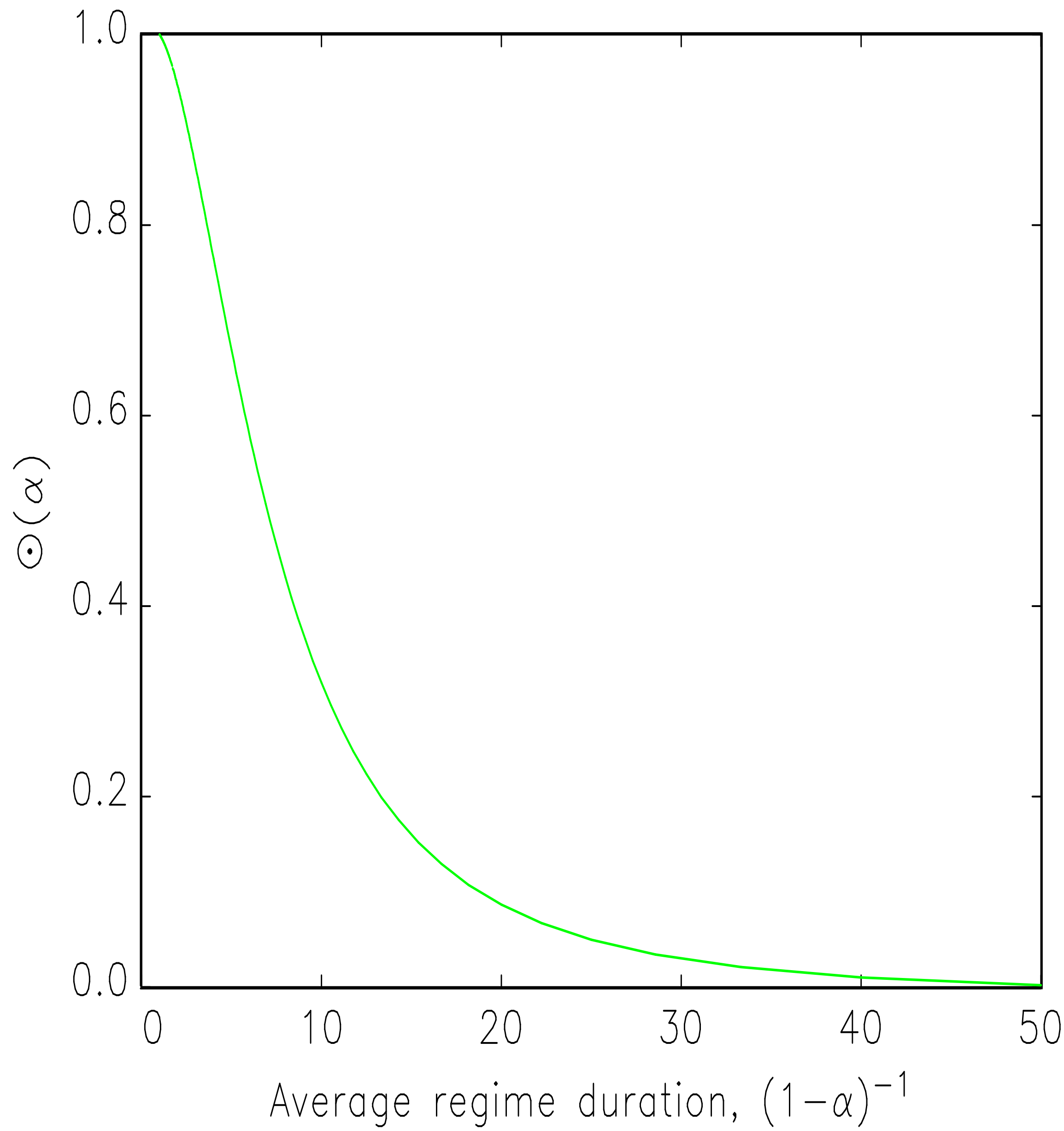

Figure 1. Expected additional gain to credibility as a function of credibility

In contrast Schaumburg and Tambalotti (2007), Figure 1 shows for the Smets-Wouters model that when the average regime duration is short most of the gains to increased credibility remain. Indeed, where Schaumburg and Tambalotti (2007) find for their model[2] that an average duration of just 10 quarters leaves only about 5 percent of the gains to commitment

[2]The model that Schaumburg and Tambalotti (2007) analyze is a simple stylized New Keynesian model consisting of the New Keynesian Phillips curve with i.i.d. markup shocks in which the output gap is the policy instrument.

outstanding, for the Smets-Wouters model Figure 1 shows that a similar performance requires an average regime duration of about 25 quarters.

6 Robust policymaking with imperfect credibility

Where the analysis in Section 5 focused on the polar cases of commitment and discretion, we now turn to consider the effects of imperfect credibility and robustness on policymaking. Although there are six shocks in the model, in our analysis here we focus on the effects of shocks to the price markup and to aggregate technology. In light of its policy objectives, the central bank always offsets the effects of the shock to the neutral interest rate, and the qualitative story that emerges regarding the effects of robustness and imperfect credibility on policymaking is consistent across the other three shocks.

6.1 The effects of imperfect credibility and robustness on the response to price-markup shocks

For a range of assumptions about credibility, Figure 2 displays the responses of inflation, the output gap, and the nominal interest rate to a one-standard-deviation price-markup shock. We focus on two types of impulse responses. The first type of impulse responses we denote as "within regime" responses. These responses are equivalent to the Type I responses of Schaumburg and Tambalotti (2007); they are constructed under the assumption that today's policymaker remains the decisionmaker in all future periods and characterize the responses within a policymaker's tenure. Following Bodenstein, Hebden, and Nunes (2010) these responses can be interpreted as the forward-guidance released by the central bank, here released in the form of a state-contingent forecast that is conditioned upon a specific shock and upon ongoing tenure. The second type of impulse responses we denote as "expected" responses. These responses are equivalent to the Type III responses of Schaumburg and Tambalotti (2007) and they characterize the responses taking into account the fact that the tenure of today's policymaker will stochastically terminate.[3]

The panels in the left-most column of Figure 2 (panels A, D, and G), show the (nonrobust) expected responses under commitment[4] and discretion and the within-regime and expected responses under imperfect credibility ($\alpha = 0.75$). In response to the price markup shock,

[3] Thus, where the Type I responses are conditioned upon a specific future sequence $\{\eta_s = 1\}_{s=t+1}^{\infty}$, the Type III responses take all possible future sequences into account.

[4] Of course, for a central bank that can commit the announced responses and the expected responses coincide.

inflation rises (panel A) and a negative output gap opens up (Panel D). The negative output gap is somewhat larger when policy is conducted under commitment than under discretion, a consequence of the discretionary stabilization bias, which leads the discretionary policy-maker to inefficiently trade-off movements in the output gap and inflation. Looking now at the equilibrium responses according to the within regime policy, with $\alpha = 0.75$ the central bank announces a policy rule that implies that while its tenure continues it will implement a policy that tightens less rapidly (Panel G), but keeps interest rates higher for longer than the commitment policy. Thus, in order to stabilize inflation the imperfectly credible central bank attempts to leverage the credibility it has by announcing that it will implements a policy during its tenure that is tighter for longer than the commitment policy. This result is consistent with Bodenstein, Hebden, and Nunes (2010), who also found that imperfectly credible central banks seek to leverage their credibility by making more extreme within-regime policy announcements. Interestingly, the expected policy looks qualitatively and quantitatively much more like the discretionary policy than the commitment policy, with the shock leading to a permanent increase in the price level.

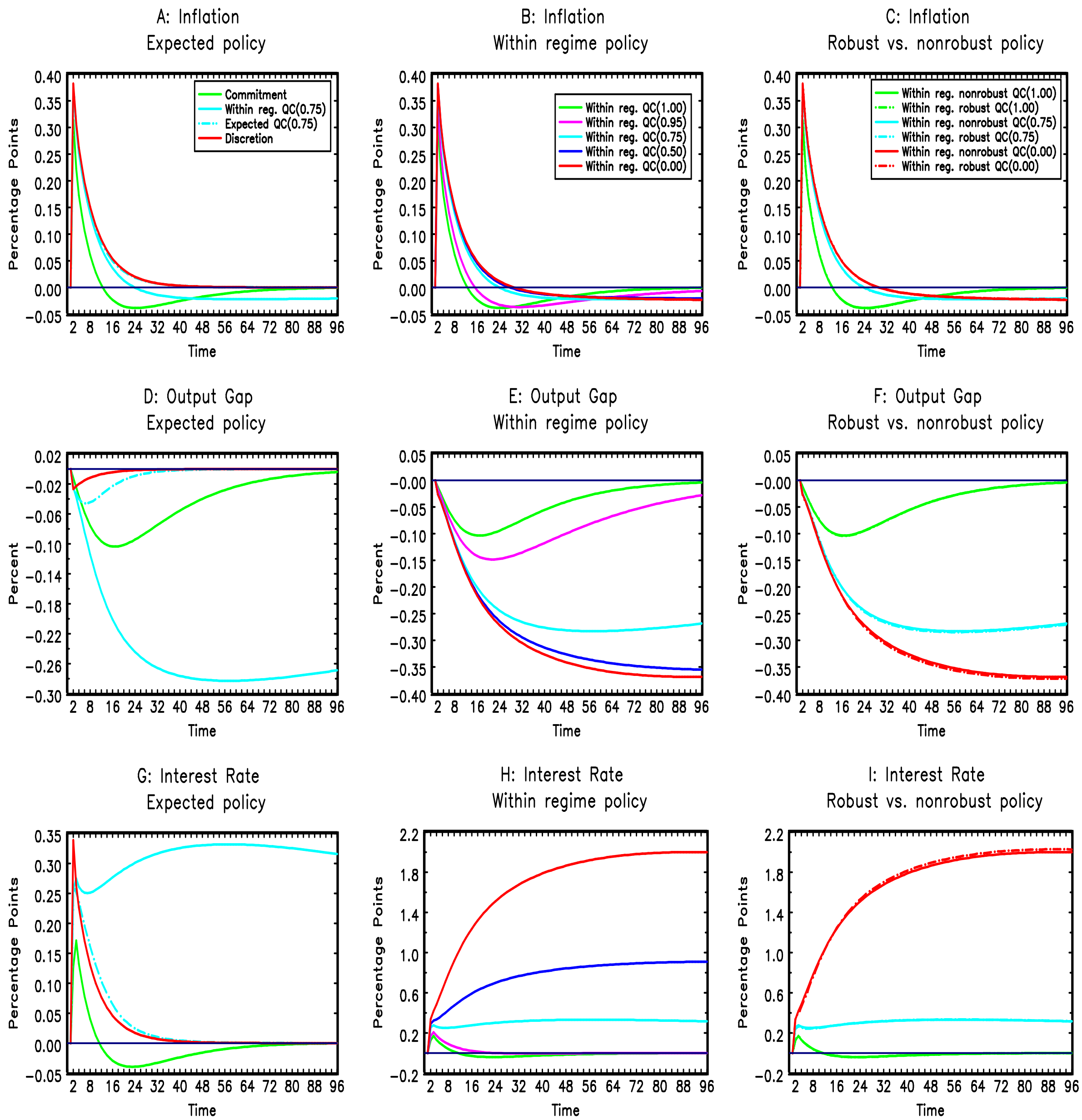

Figure 2. Responses to a 1-s.d. price-markup shock (μ=0.25)

Having seen how imperfect credibility drives a wedge between the within regime responses and the expected responses, we now investigate in more detail the effects that imperfect credibility has on the within-regime policy. To this end, the panels in the middle column of Figure 2 (panels B, E, and H) display, for varying values of α, the within-regime policy responses following a one-standard deviation price markup shock. When $\alpha = 1$, and the central bank is perfectly credible, the impact effect of the markup shock is to raise inflation (panel B) and lower the output gap (panel E). Because the central bank is committed to returning inflation to target (here the inflation target is a rate of zero), the commitment policy anchors long-run inflation expectations firmly on the target. With inflation expectations anchored on the target, monetary policy in the short-run can be directed at stabilizing the output gap. According to the commitment policy, the central bank raises interest rates in the short-run (panel H) while promising to subsequently lower rates as inflation declines. When $\alpha = 0$, and the central bank has no credibility, the within-regime responses are extreme, but are implemented with zero-probability. When the central bank has imperfect credibility the announced within-regime responses suggest an extreme tightening of policy, much like discretionary behavior, except when α is close to one.

Lastly, the panels in the third column of Figure 2 (panels C, F, and I) illustrate the effect that the central bank's concern for robustness has on its within-regime responses. With the robustness parameter, θ, chosen such that $\omega = \overline{\omega}$, the main results that emerge are the following. First, for this model robustness has only very small effects on the within-regime responses. The effects of credibility on policy are quantitatively much more important than those of robustness. Second, the within-regime responses associated with a perfectly-credible central bank are essentially unaffected by the central bank's desire for robustness.

6.2 The effects of imperfect credibility and robustness on the response to technology shocks

Figure 3 displays the responses of inflation, the output gap, and the nominal interest rate to a one-standard-deviation aggregate-technology shock. Although the model, obviously, behaves quite differently following aggregate technology shocks than it does following price-markup shocks, the conclusions regarding the qualitative effects of imperfect credibility and robustness are similar. Looking that the responses associated with the perfectly credible central bank ($\alpha = 1$), because a rise in aggregate technology allows more goods to be produced from a given set of inputs the effects of the shock are to raise output and lower inflation (panel A). Because the central bank's policy objective function assigns a large relative weight to stabilizing inflation (recall, $\mu = 0.25$), the effect of the technology shock on monetary policy is to lower the nominal interest rate (panel G). With the nominal interest rate declining more that one-for-one with inflation, the real interest rate declines and this stimulates aggregate demand, opening up a positive output gap (panel D) and creating upward pressure on inflation. As inflation rises, monetary policy begins to tighten and the positive output gap begins to close.

The panels in the left-most column of Figure 3 (panels A, D, and G) reveal that the imperfectly credible central bank ($\alpha = 0.75$) responds to the shock with a within-regime policy that is more extreme than that of the perfectly-credible central bank. Indeed, the imperfectly credible central bank sees inflation systematically below target and announces a policy path that has low interest rates for a considerable period. At the same time, the responses to the technology shock are all relatively small, a consequence of the fact that with a policy objective directed at stabilizing the economy about its flex-price equilibrium, policy largely accommodates shocks to technology. The panels in the middle column of Figure 3 (panels B, E, and H) show that changes in credibility have relatively muted effects on both inflation and the output gap while having larger effects on the within-regime path for the interest rate. The panels in the third column of Figure 3 (panels C, F, and I) further highlight that the central bank's desire for robustness has negligible effect on the within-regime responses associated with a perfectly-credible central bank, while having a larger, but still small, effect on the within-regime policy pursued by the discretionary central bank.

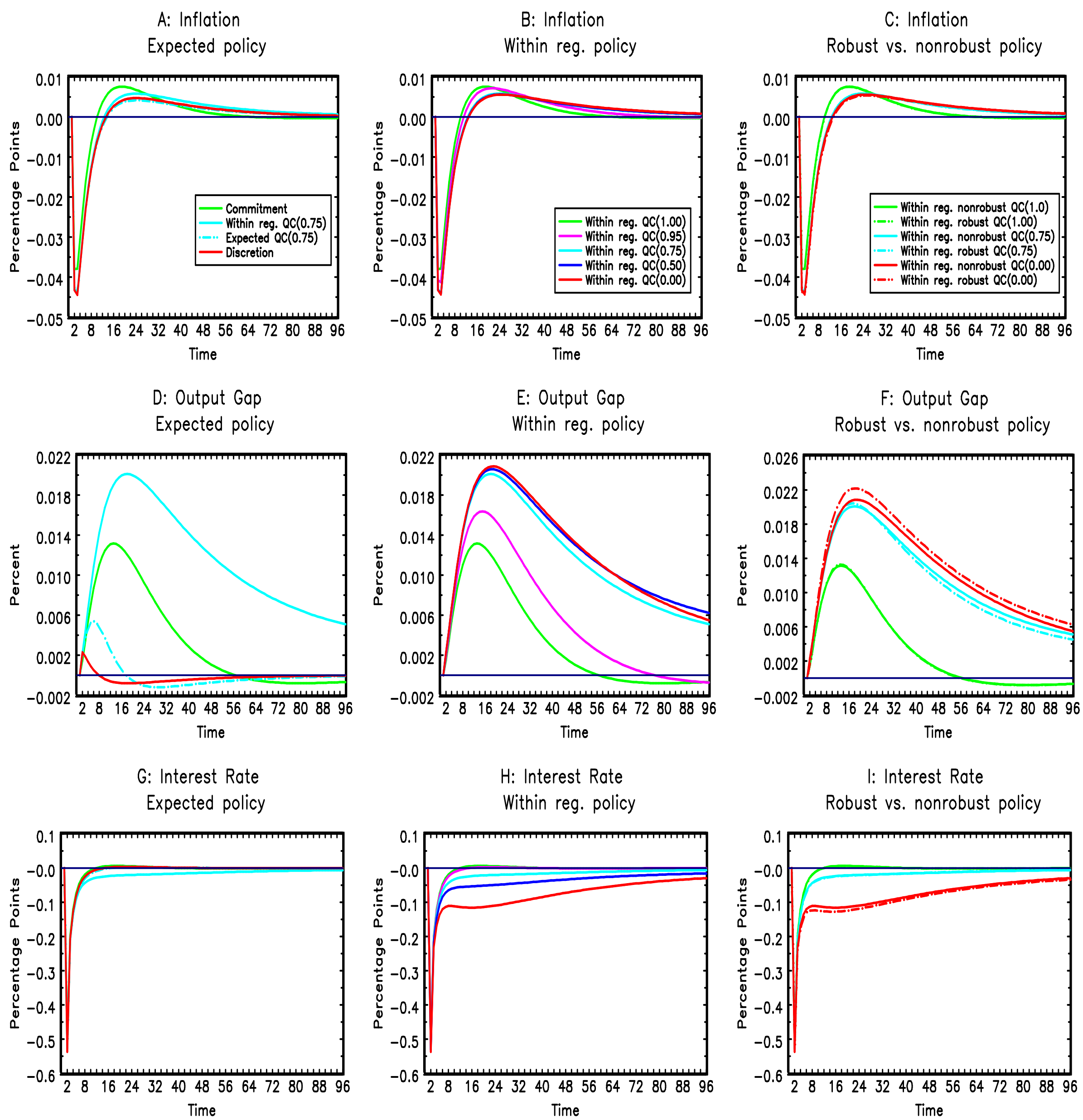

Figure 3. Responses to a 1-s.d. aggregate technology shock (μ=0.25)

6.3 Is robustness a substitute for credibility?

It is well-known that commitment is superior to discretion and that higher credibility leads to better economic outcomes when the model is correctly specified. However, when the central bank has doubts about its model and implements a policy that robust in the Hansen-Sargent sense, then the model's equilibrium with robust policy (the approximating equilibrium) will differ from the model's equilibrium with nonrobust policy (the rational expectations equilibrium). This raises the question of whether the central bank's desire for robustness produces an improvement or a deterioration in policy performance when the central bank has imperfect credibility. Is the policy performance, measured according to the policy objective function (equation 27), associated with the robust policy higher or lower than that associated with the nonrobust policy and how is the relative performance of these two policies affected by credibility? The analysis in this section relates to Dennis (2010) who used a stylized medium-scale New Keynesian DSGE model to show that a central bank's desire for robustness could, in principle, lead to improved outcomes when policy is conducted under discretion. Here we expand on Dennis (2010) by examining this issue in the Smets and Wouters model and by considering imperfect credibility rather than just discretion.

For the Smets and Wouters (2007) model, Figure 4 examines the relationship between relative policy performance and robustness, θ, for varying levels of credibility, α. If robust-loss relative to nonrobust-loss $\left(V^{qc}\left(\alpha,\theta\right)/V^{qc}\left(\alpha,\infty\right)\right)$ is greater than one, then robustness leads to a deterioration in policy performance. Alternatively, if robust-loss relative to nonrobust-loss is less than one, then robustness leads to an improvement in policy performance.

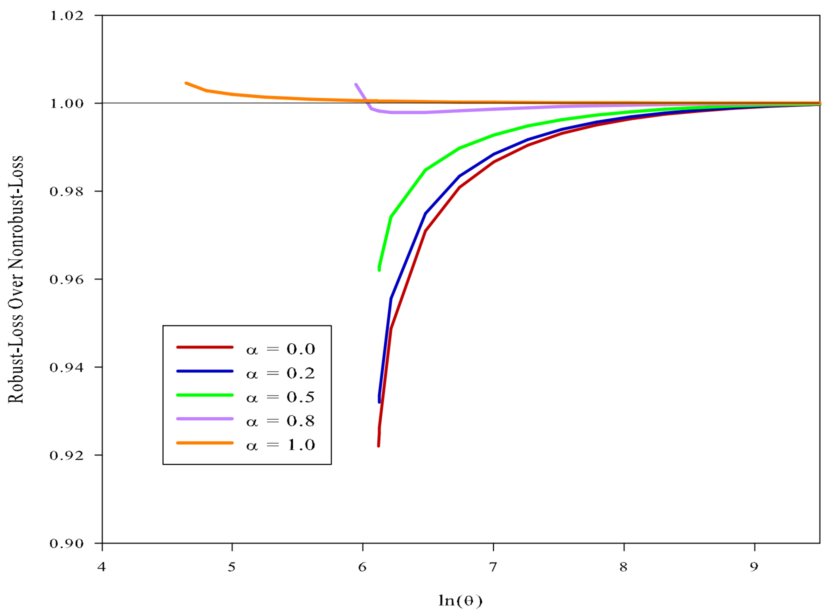

Figure 4. The effectiveness of robustness as a substitute for credibility (μ=0.25)

With higher values of θ associated with less concern for model misspecification, and with policy performance for a given θ measured relative to the benchmark in which the model is known to be correctly specified, relative policy performance $\left(V^{qc}(\alpha,\theta)/V^{qc}(\alpha,\infty)\right)$ converges to one as θ rises to infinity for all levels of credibility. Figure 4 shows that whether the equilibrium outcomes associated with the approximating equilibrium are superior or inferior to those associated with the nonrobust equilibrium depends on whether the central bank can commit. When credibility is perfect ($\alpha = 1$) the central bank's desire for robustness leads to a slight decline in relative policy performance. Similarly, when the central bank has zero-credibility ($\alpha = 0$) the central bank's desire for robustness improves relative policy performance. Each of these findings is consistent with Dennis (2010), and establishes for the Smets and Wouters (2007) model that a central bank's desire for robustness can serve somewhat like a commitment mechanism even when policy is conducted under discretion. In addition, Figure 4 shows that a desire for robustness can act as a substitute for credibility and improve relative policy performance for most levels of credibility, and not just for pure discretion ($\alpha = 0$). Indeed, only when credibility is very high—close to perfect—does the central bank's desire for robustness ever worsen policy performance.

6.4 How detectable are the specification errors?

Our analysis of robust policymaking has assumed that the robustness parameter, θ, equals the threshold value, $\underline{\theta}$. This assumption expresses the idea that the central bank is as concerned as it can be about the approximating model while still holding the view that the approximating model is a useful approximation of the actual data-generating process. A consequence of this assumption is that the effects of robustness on the impulse responses shown in Figures 2 and 3 cannot be made more damaging through a different—and more pessimistic—choice of θ. The fact that the effects of model misspecification appear small even with $\theta = \underline{\theta}$ may well imply that the Smets and Wouters (2007) model can be destabilized by relatively small specification errors, even under a robust policy rule. Such a result would not be unexpected because it has been shown elsewhere that the performance of optimal policy rules, which exploit fully a model's structure, can be very poor when that structure is incorrect (Levin, Wieland, and Williams, 2003), providing a popular argument for the use of optimized simple rules, which exploit less structure (McCallum, 1988). With these issues in mind, here we ask the question of whether the central bank is likely to be able to detect the specification errors and how their detection is affected by imperfect credibility.

To explore this question we employ the notion of a detection-error probability that is promoted in a series of papers by Hansen and Sargent (see, for example, Anderson, Hansen, and Sargent, 2003). A detection-error probability is the probability that an econometrician observing equilibrium outcomes would make an incorrect inference about whether the approximating equilibrium or the worst-case equilibrium generated the data. The intuitive connection between θ and the probability of making a detection error is that when θ is small, greater differences between the distorted model and the approximating model (more severe misspecifications) can arise, which are more easily detected. In its top panel, Figure 5 displays the relationship between the (log of the) robustness parameter θ and the probability of making a detection error for discretion ($\alpha = 0.00$), perfect credibility ($\alpha = 1.00$), and imperfect credibility ($\alpha = 0.50$). In its bottom panel, Figure 5 displays the relationship between the robustness parameter, θ, and the distortion budget, ω, calculated according to equation (8).

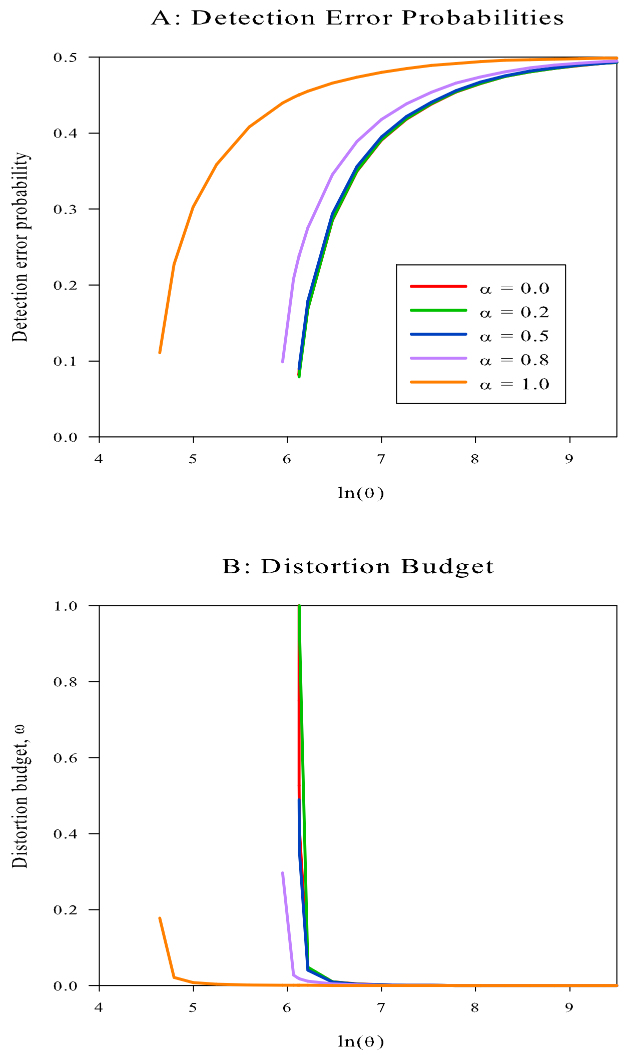

Figure 5. The relationship between detectability, the distortion budget, and credibility (μ=0.25)

The following main results emerge from Figure 5. First, the detection error probabilities and the distortions budgets associated with imperfect credibility are very similar to those for discretion, except when α is close to one. Second, although the detection error probabilities are similar at the breakdown points, $\omega = \overline{\omega}(\alpha)$, these breakdown points are associated with very different distortion budgets. In particular, the distortion budget for $\alpha = 1$ is the smallest of those considered, suggesting that commitment policies are indeed more susceptible to model misspecification. Together the two panels in Figure 5 suggest that the commitment policy is more fragile than the quasi-commitment policies, breaking down with smaller specification errors, that the quasi-commitment policies behave similarly to the discretionary policy, and that the distortion budget is larger under discretion, which is consistent with the robustness results in Figures 2 and 3.

6.5 The effect of a greater weight on output stabilization

The results above were obtained under the maintained assumption that the relative weight assigned to output stabilization in the policy objective function is $\mu = 0.25$. To assess whether our results are qualitatively sensitive to this assumed value for μ we repeated the analysis, but under the maintained assumption that $\mu = 4.00$, i.e., that the weight assigned to output stabilization is four times the weight assigned to inflation stabilization. Although the nature of the impulse response functions do change, simply reflecting the greater importance that the central bank places on output stabilization, the qualitative results do not change. With the robustness parameter set to its threshold value, the effects of robustness on the impulse response functions is relatively small, considerably smaller than the effects of imperfect credibility. Further, the central bank's lack of credibility continues to motivate it to leverage what credibility it has by seeking to implement a within-regime policy response that is more extreme than that associated with perfect credibility. In addition, the central bank's desire for robustness continues to generate improved policy performance, except when credibility is close to perfect.

7 Conclusion

This paper has considered the decision problem facing an imperfectly credible central bank that seeks to conduct monetary policy using a model whose structure it has doubts about. Motivating this study is the increased use by central banks of policy announcements in the form of model-based forecast-contingent forward-guidance about future policy. In this paper, the central bank's doubts about its model are modeled via the robust control literature, giving rise to a maxmin problem as per Hansen and Sargent (2008), while imperfect credibility is modeled according to the quasi-commitment literature. The resulting decision problem allows us to study separately, and in combination, the effects that robustness and imperfect credibility have on central bank behavior and economic outcomes. Usefully, this decision problem accommodates commitment, discretion, quasi-commitment, robust control, and nonrobust control as special cases.

With the Smets and Wouters (2007) model providing the laboratory, our examination of robust policymaking with imperfect credibility offers the following main findings. First, a central bank's credibility gives it a powerful lever for managing private-sector expectations and for stabilizing the economy. The importance of credibility for outcomes is manifest in the magnitude of the discretionary stabilization bias and in the finding that short average regime durations leave most of the gap between the discretionary policy and the commitment policy unclosed. Related to these findings, in contrast to Schaumburg and Tambalotti (2007), we find that even relatively small departures from perfect credibility produce big declines in policy performance, giving rise to a form of discretionary stabilization bias. Second, consistent with Bodenstein, Hebden, and Nunes (2010), a consequence of imperfect credibility is that it can give a central bank an incentive to issue what may appear to be an extreme within-regime policy response in an effort to leverage what credibility it has. Third, to the extent that robustness is important for how policy responds to shocks, it appears to be more important for low-credibility central banks. In particular, a low-credibility central bank can benefit from a desire for robustness with this desire acting somewhat as a substitute for credibility.

A Appendix: Risk-sensitive preferences and robust control with quasi-commitment

We showed in Section 3 that the solution to the robust-control problem under discretion could equivalently be obtained from a formulation with risk-sensitive preferences. In this appendix we extend that result to establish a risk-sensitive formulation that is equivalent to the robust-control decision problem with quasi-commitment. One simplifying assumption that we make is that $\mathbf{A}_0$ has full rank. Without loss of generality, then, we assume that $\mathbf{A}_0 = \mathbf{I}$. With this assumption, and assuming $\alpha \in (0,1]$ (ruling out the discretionary case), the constraints according to the approximating model can be written as

$$\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \mathbf{B}\mathbf{u}_t + \mathbf{C}\varepsilon_{\mathbf{x}t+1}, \tag{A1}$$

while those according to the distorted model can be written as

$$\mathbf{z}^*_{t+1} = \mathbf{A}\mathbf{z}_t + \mathbf{B}\mathbf{u}_t + \mathbf{C}\left(\mathbf{v}_{t+1} + \varepsilon_{\mathbf{x}t+1}\right), \tag{A2}$$

where $\mathbf{z}_t = \begin{bmatrix} \mathbf{x}_t' & \mathbf{y}_t' \end{bmatrix}'$. To show the connection between the robust-control problem and the risk-sensitive preferences problem it is convenient to utilize the solution strategy of Backus and Driffill (1986) which begins by treating $\mathbf{z}_t$, which contains nonpredetermined variables, as the state vector. Accordingly, the risk-sensitive preferences formulation takes the form

$$\mathbf{z}_t'\mathbf{P}\mathbf{z}_t + p = \min_{\mathbf{u}_t}\left[L\left(\mathbf{z}_t, \mathbf{u}_t\right) - \frac{1}{\sigma}\ln\left(\mathrm{E}_t\left(\exp\left(\sigma\left(\mathbf{z}_{t+1}'\widetilde{\mathbf{P}}\mathbf{z}_{t+1} + p\right)\right)\right)\right)\right], \tag{A3}$$

where $\sigma \leq 0$,

$$\widetilde{\mathbf{P}} = \alpha \mathbf{P} + (1-\alpha)\, \mathbf{S}'\mathbf{S}'^{-1}\mathbf{P}\mathbf{S}^{-1}\mathbf{S},$$

with $\mathbf{S} = \left[\begin{array}{cc} \mathbf{I} & \mathbf{0} \end{array}\right]$, and the constraints are given by equation (A1). Employing Jacobson (1973), equation (A3) can be expressed as

$$\mathbf{z}_t'\mathbf{P}\mathbf{z}_t + p = \min_{\mathbf{u}_t} \left[L(\mathbf{z}_t, \mathbf{u}_t) + \beta \mathbf{E}_t \left(\mathbf{z}_{t+1}' D\left(\widetilde{\mathbf{P}}\right) \mathbf{z}_{t+1} + \widehat{p} \right) \right], \tag{A4}$$

where

$$D\left(\widetilde{\mathbf{P}}\right) = \widetilde{\mathbf{P}} - \sigma \widetilde{\mathbf{P}}\mathbf{C}\left(\mathbf{I} + \sigma \mathbf{C}'\widetilde{\mathbf{P}}\mathbf{C}\right)^{-1}\mathbf{C}'\widetilde{\mathbf{P}}.$$

In contrast, the robust control formulation of the decision problem is

$$\mathbf{z}_t'\mathbf{P}\mathbf{z}_t + p = \min_{\mathbf{u}_t} \max_{\mathbf{v}_{t+1}} \left[L(\mathbf{z}_t, \mathbf{u}_t) - \beta\theta \mathbf{v}_{t+1}'\mathbf{v}_{t+1} + \beta \mathrm{E}_t \left(\mathbf{z}_{t+1}^{*\prime} \widetilde{\mathbf{P}} \mathbf{z}_{t+1}^{*} + p \right) \right], \tag{A5}$$

where $\theta \geq 0$ and the constraints are given by equation (A2). Performing the inner maximization yields

$$\mathbf{v}_{t+1} = \mathrm{E}_t \left[\frac{1}{\theta} \left(\mathbf{I} - \frac{1}{\theta} \mathbf{C}'\widetilde{\mathbf{P}}\mathbf{C} \right)^{-1} \mathbf{C}'\widetilde{\mathbf{P}}\mathbf{z}_{t+1} \right]. \tag{A6}$$

Substituting equations (A6) and (A2) back into equation (A5) gives

$$\mathbf{z}_t'\mathbf{P}\mathbf{z}_t + p = \min_{\mathbf{u}_t} \left[L(\mathbf{z}_t, \mathbf{u}_t) + \beta \mathrm{E}_t \left(\mathbf{z}_{t+1}' \left[\widetilde{\mathbf{P}} + \frac{1}{\theta} \widetilde{\mathbf{P}}\mathbf{C} \left(\mathbf{I} - \frac{1}{\theta}\mathbf{C}'\widetilde{\mathbf{P}}\mathbf{C} \right)^{-1} \mathbf{C}'\widetilde{\mathbf{P}} \right] \mathbf{z}_{t+1} + \overline{p} \right) \right], \tag{A7}$$

where the constraints are now given by the approximating mode, equation (A1).

With $\theta = -\sigma^{-1}$ the solutions to equations (A4) and (A7) lead to the same decision rule

$$\mathbf{u}_t = \mathbf{F}\mathbf{z}_t.$$

Following Backus and Driffill (1986) the next step is to transform the solution from one depending on predetermined and nonpredetermined variables to one depending on predetermined and costate variables, where the latter are the analogue of the multipliers $\mathbf{\Xi}_{t-1}$ in the text. Accordingly, let

$$\mathbf{\Xi}_{t-1} = \mathbf{P}_{21}\mathbf{x}_t + \mathbf{P}_{22}\mathbf{y}_t,$$

where $\mathbf{P}_{21}$ and $\mathbf{P}_{22}$ are submatrices of $\mathbf{P}$. Then the solution has

$$\begin{aligned} \mathbf{H} &= -\mathbf{P}_{22}^{-1}\mathbf{P}_{21}, \\ \mathbf{C}_2 &= -\mathbf{P}_{22}^{-1}\mathbf{P}_{21}\mathbf{C}_1. \end{aligned} \tag{A8}$$

Note that substituting equation (A8) into equation (A6) leads to

$$\mathbf{v}_{t+1} = \mathrm{E}_t \left[\frac{1}{\theta} \left(\mathbf{I} - \frac{1}{\theta} \mathbf{C}_1' \left(\mathbf{P}_{11} - \mathbf{P}_{21}'\mathbf{P}_{22}^{-1}\mathbf{P}_{21} \right) \mathbf{C}_1 \right)^{-1} \mathbf{C}_1' \left(\mathbf{P}_{11} - \mathbf{P}_{21}'\mathbf{P}_{22}^{-1}\mathbf{P}_{21} \right) \mathbf{x}_{t+1} \right],$$

which further establishes that the worst-case specification errors depend only on the the expected future predetermined variables (Dennis, 2008).

B Appendix: Detection-error probability

Let A denote the approximating model and B denote the worst-case model; then, assigning equal prior weight to each model and assuming that model selection is based on the likelihood ratio principle, Hansen, Sargent, and Wang (2002) show that detection-error probabilities are calculated according to

$$p(\theta) = \frac{\text{prob}(A|B) + \text{prob}(B|A)}{2},$$

where prob$(A|B)$ (prob$(B|A)$) represents the probability that the econometrician erroneously chooses A (B) when B (A) generated the data. Let $\{\mathbf{z}_t^B\}_1^T$ denote a finite sequence of economic outcomes (the shocks, the shadow prices, the endogenous variables, and the followers' and leader's decision variables) generated by the worst-case equilibrium, and let L_{AB} and L_{BB} denote the likelihood associated with models A and B, respectively; then the econometrician chooses A over B if $\log(L_{BB}/L_{AB}) < 0$. Generating M independent sequences $\{\mathbf{z}_t^B\}_1^T$, prob$(A|B)$ can be calculated according to

$$\text{prob}(A|B) \approx \frac{1}{M}\sum_{m=1}^{M} \text{I}\left[\log\left(\frac{L_{BB}^m}{L_{AB}^m}\right) < 0\right],$$

where $\text{I}[\log(L_{BB}^m/L_{AB}^m) < 0]$ is the indicator function that equals one when its argument is satisfied and equals zero otherwise; prob$(B|A)$ is calculated analogously using data generated from the approximating model.

Let

$$\begin{aligned}\mathbf{z}_{t+1} &= \mathbf{H}_A\mathbf{z}_t + \mathbf{G}\boldsymbol{\varepsilon}_{t+1}\\ \mathbf{z}_{t+1} &= \mathbf{H}_B\mathbf{z}_t + \mathbf{G}\boldsymbol{\varepsilon}_{t+1}\end{aligned}$$

govern equilibrium outcomes under the approximating equilibrium and the worst-case equilibrium, respectively. Using the Moore-Penrose inverse,

$$\widehat{\boldsymbol{\varepsilon}}_{t+1}^{i|j} = \left(\mathbf{G}'\mathbf{G}\right)^{-1}\mathbf{G}'\left(\mathbf{z}_{t+1}^j - \mathbf{H}_i\mathbf{z}_t^j\right), \quad \{i,j\} \in \{A,B\}$$

are the inferred innovations in period $t+1$ when model i is fitted to data $\{\mathbf{z}_t^j\}_1^T$ generated from model j, and let $\widehat{\boldsymbol{\Sigma}}^{i|j}$ be the associated estimates of the innovation variance-covariance matrices. Note that the Moore-Penrose inverse picks out the shock process from among the variables in $\mathbf{z}_t$.

Assuming that the innovations are normally distributed, it is easy to show that

$$\begin{aligned}\log\left(\frac{L_{AA}}{L_{BA}}\right) &= \frac{1}{2}\text{tr}\left(\widehat{\boldsymbol{\Sigma}}^{B|A} - \widehat{\boldsymbol{\Sigma}}^{A|A}\right)\\ \log\left(\frac{L_{BB}}{L_{AB}}\right) &= \frac{1}{2}\text{tr}\left(\widehat{\boldsymbol{\Sigma}}^{A|B} - \widehat{\boldsymbol{\Sigma}}^{B|B}\right).\end{aligned}$$

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