International Output Convergence: Evidence from an AutoCorrelation Function Approach*

Giovanni Caggiano†
University of Glasgow

Leone Leonida
Queen Mary University of London

Abstract

This paper uses an AutoCorrelation Function approach to develop new tests for international output convergence. Using per capita GDP for 15 OECD countries observed over a century, we find that the hypothesis of conditional convergence is unsupported; that, the United States apart, the linearized neoclassical growth model fails to replicate the transitional dynamics of OECD economies; and that these economies do not behave like a club.

Keywords: Autocorrelation Function; Convergence; Neoclassical Growth Model.

JEL: C22; N10; O40.

*A previous version of this paper, circulated as "Testing for Convergence Processes in an AutoCorrelation Function Framework", has been presented at the Xth Spring Meeting of Young Economists (SMYE), Geneva, Switzerland, April 22-24, 2005, at the 21st Annual EEA congress, Vienna, August 24-28, 2006, the University of Lecce and the University of Brescia. We thank all participants, George Kapetanios and Campbell Leith for helpful comments. We are especially indebted to Gregg Huff for a thorough reading of a previous version of the paper. The usual disclaimer applies.

†Corresponding author. Department of Economics, University of Glasgow, Glasgow, G12 8RT, UK. Tel.: +44-141-3305059; fax: +44-141-3304940. E-mail address: g.caggiano@lbss.gla.ac.uk.

1 Introduction

This paper develops new definitions and tests for convergence in international output by using an AutoCorrelation Function (ACF, hereafter) approach. Such an approach encompasses both exponential and fractional $\beta-$convergence as special cases. It also allows for nonlinear transitional dynamics to the equilibrium path, approximated with a time trend as implied by the neoclassical model of growth and as standard in the time series literature on convergence. We examine the statistical properties of detrended real output per capita by estimating its ACF. Because it is a measure of the correlation of the series with its past, one can interpret the ACF of detrended output in terms of the transitional dynamics of the economy to its steady state path: once the economy departs from it, the time required for the ACF to go to zero is a measure of the time required for the economy to return to its long run equilibrium path. Convergence requires that departures from the equilibrium must be temporary, implying that the ACF becomes zero in the observed sample.

The approach we propose is novel in many respects. First, it encompasses both the unit root and the fractional integration frameworks as special cases. In terms of the ACF, the rejection of a unit root in detrended output (the standard case of exponential $\beta-$convergence) implies an exponential rate of decay. The case of no convergence, a unit root in detrended output, is equivalent to a never-decaying ACF. The case of fractional $\beta-$convergence implies an ACF that decays slowly, at an hyperbolic rate. Allowance for fractional rates of decay represents a useful generalization of the strict dichotomy (exponential rate of convergence versus no convergence) implied by unit root tests. However, it does not include the possibility of a slow and non-monotonic rate, which would be reflected in an ACF that is not strictly convex. In this case, one can still find evidence of convergence, but with a non-standard transitional path that a testing procedure based on linear processes may not capture. Second, an ACF framework can be used to develop new time series definitions and tests of conditional and unconditional β-convergence, in the spirit of Bernard and Durlauf (1995). In terms of the empirical ACF of detrended output, convergence requires that the sample ACF becomes statistically insignificant after a finite number of lags. Third, the approach can be used to test whether the log-linearized Solow growth model replicates the transitional dynamics of an economy.

We use GDP per capita of 15 OECD economies observed over a century and find a number of interesting results. First, with the exception of the United States, exponential $\beta-$convergence can always be rejected. For some economies, we find that a non-standard process of conditional convergence takes place, at a slow rate and with a non-monotonic, persistent pattern. For most economies, however, we find no conditional convergence. Second, as an implication of the previous results, we find that the log-linearized version of the Solow model fails to replicate the transitional dynamics of 14 of the 15 OECD economies examined, the United States again being the exception. Finally, we do not find unconditional convergence among OECD economies, which is against the claim that they behave like a club.

The structure of the paper is as follows. Section 2 examines the meaning of convergence in a time series framework. Section 3 describes the data. Section 4 outlines our testing procedure. Section 5 presents our empirical results, and Section 6 concludes.

2 Convergence in a Time Series Framework

The debate over output convergence has been central in the empirical literature on growth for the last two decades. Seminal papers by Baumol (1986) and Abramovitz (1986), and the challenging evidence of Barro and Sala-i-Martin (1992) that economies are converging at a constant speed, initiated ongoing debates on whether countries converge to their own steady state path and whether countries tend to converge to a common steady state path, the conditional and unconditional convergence hypothesis respectively.

Early studies on convergence were based on a regression of a country's growth rate on a set of regressors including some determinants of its equilibrium level, which can be derived either from a formal model of growth (Mankiw, Romer and Weil, 1992) or not (Abramovitz, 1986). In this framework, the test for convergence adds a proxy of the country's initial wealth to the set of regressors. A negative and significant coefficient associated with initial wealth would be interpreted as evidence for convergence, since it implies that richer economies grow, on average, slower than poorer countries.

Some researchers (Levine and Renelt, 1992; Islam, 2003; Quah 1996) suggest, however, that this approach may be misleading. Two critiques seem particularly relevant: first, as long as the marginal productivity of capital is decreasing, it is possible for a mixed set of converging and diverging economies that the estimated coefficient associated with the gap variable is negative and significant; second, the short-run transitional dynamics and the long-run steady-state behavior cannot be disentangled in a cross section regression (Bernard and Durlauf, 1996). Panel regressions, although providing more precise estimates of the structural parameters of interest, do not address these problems (Durlauf and Quah, 1999).

To overcome the drawbacks associated with the use of cross-sectional data, Bernard and Durlauf (1995) suggest testing for convergence in a time series environment. Let $y_{i,t}$ denote the log of real per capita output in country i, $y_{i,t} = \ln(Y_{i,t}/L_{i,t})$, and $y^*_{i,t}$ its steady state value at time t. Then, the following model provides a simple and intuitive framework to test for convergence:

$$y_{i,t} - y^*_{i,t} = (1 - \beta_i)\left(y_{i,t-1} - y^*_{i,t-1}\right). \tag{1}$$

If β_i is zero in equation (1), the gap between the logarithm of real GDP per capita and its steady state value does not dissipate over time. There is, in other words, no tendency for country i's real per capita GDP to converge to its own steady state path. Alternatively, if β_i is positive and less than one, country i's real per capita GDP tends to its steady state level at an exponential $\beta_i\%$ rate.

In this framework, testing for convergence of an economy to its own steady state position (if $y^*_{i,t}$ denotes country i's specific time trend), or convergence of one economy to another (if $y^*_{i,t}$ denotes real per capita GDP of country r, $y_{r,t}$) or convergence of a set of economies to a common trend (if $y^*_{i,t}$ denotes a common trend, estimated by using all economies $i = 1, \ldots, N$, $\overline{y}_t$), can be done by testing the hypothesis of no unit root in the data generating process of $\left\{y_{i,t} - y^*_{i,t}\right\}$, $\{y_{i,t} - y_{r,t}\}$ and $\{y_{i,t} - \overline{y}_t\}$ respectively.

Although inference based on unit root tests is widely used, it is generally acknowledged that these tests are inconclusive in finite samples because of their low power. Some authors (see, among others, Diebold and Rudebusch, 1989; Gil-Alãna and Robinson, 1997; Abadir and Taylor, 1999) argue that the contradictory results obtained by using unit root tests to determine the properties of real GDP per capita can be explained in terms of fractional integration. If real GDP per capita is a fractionally integrated process, then convergence to a country-specific or to a common, equilibrium path would not take place at an exponential rate but at a much slower hyperbolic rate, which tests for unit roots would not capture. Michelacci and Zaffaroni (2000) apply tests for fractional integration to examine convergence processes and find evidence of (fractional) $\beta-$convergence for all the OECD economies included in their dataset, consistent with a constant 2% speed of convergence.

Even though it has been argued that Michelacci and Zaffaroni's findings are not robust to changes in the estimation procedure (Silverberg and Vespargen, 2000), we believe that this work represents a valuable step towards a correct understanding of long run convergence processes. Estimates of the memory parameter can shed light on whether real GDP reverts to its steady-state path at a hyperbolic rather than at an exponential rate. However, the dynamics implied by fractionally integrated processes can account only for strictly convex hyperbolic rates of decay for the ACF. This represents no loss of information if the pattern of the transitional dynamics is monotonic, that is, if real output reverts to its trend from above (below) in response to a positive (negative) shock, but not if the transitional pattern is non-monotonic. This pattern would be reflected in an ACF with a shape that alternates convex and concave sections, which cannot be derived from fractionally integrated models.

3 Data

Our data set includes real GDP per capita for 15 OECD countries (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Sweden, UK and USA) over the period 1900-2000. We use the Penn World Table (MARK 6.1) and extend up to 2000 the annual log real GDP per capita series included in Bernard and Durlauf (1995). Data is spliced at 1987 as the base year.

4 Econometric Methodology

We examine the time series properties of detrended GDP per capita by looking at its sample autocorrelation function. Let $y_{i,t}^{d}$ be the logarithm of real GDP per capita, for country $i = 1, \ldots, N$ at time $t = 1, ..., T$, expressed as deviations from a linear trend. The ACF of $y_{i,t}^{d}$ at lag τ is given by:

$$\rho_i^d(\tau) \equiv \frac{\gamma_i(\tau)}{\gamma_i(0)} = \frac{\sum_{t=\tau+1}^{T}\left(y_{i,t}^{d} - \bar{y}_{i,t}^{d}\right)\left(y_{i,t-\tau}^{d} - \bar{y}_{i,t-\tau}^{d}\right)}{\sqrt{\sum_{t=\tau+1}^{T}\left(y_{i,t}^{d} - \bar{y}_{i,t}^{d}\right)^2 \sum_{t=1}^{T-\tau}\left(y_{i,t-\tau}^{d} - \bar{y}_{i,t-\tau}^{d}\right)^2}}, \quad (2)$$

where

$$\bar{y}_{i,t-\tau}^{d} = \frac{1}{T-\tau}\sum_{t=1}^{T-\tau} y_{i,t}^{d},$$

and

$$\bar{y}_{i,t}^{d} = \frac{1}{T-\tau}\sum_{t=\tau+1}^{T} y_{i,t}^{d}.$$

It should be noted that the formulation in (2) is different from the standard textbook version of the ACF, and allows us to account for the potential nonstationarity in the mean of the series.

In Figure 1, Panel A compares the behavior of the ACF of United States and Japanese detrended real per capita GDP, representative of two different dynamic patterns. Following the common practice of discarding a proportion of the end lags of the empirical ACF (see Box and Jenkins, 1976), we plot only 3/4 of the lags for each series. The difference in the behavior of the two ACFs is remarkable: whereas the ACF of U.S. detrended output dies out rather quickly, the ACF of Japan does not revert to zero at an exponential rate. For Japan, the effects of a shock appear very persistent, such as to affect the difference between the level of GDP and its trend for an indefinitely long period of time. Moreover, this is common to all the economies we examine: the remaining detrended GDPs are all characterized by ACFs which, as in the case of Japan, do not quickly revert to zero - see Panel B, where the bold line is the ACF of U.S.

<Insert Figure 1 here>

Testing whether the sample ACFs are statistically different, or at what lag they become statistically insignificant, must be based on a valid inference set up. To avoid making any assumption on its time series properties, one must account for the potential non stationarity of detrended GDP. A general theory for the construction of asymptotic confidence bands for the ACF of a nonstationary process does not exist in the literature and is beyond the scope of this paper. In the absence of valid theoretical results, we will confine ourselves to a method that combines resampling and subsampling techniques for dependent variables.

Politis, Romano and Wolf (2004) show that subsampling confidence intervals for the ACF are asymptotically valid even in the presence of a possible unit root in the underlying data generating process. While providing asymptotically valid confidence bands, subsampling does not allow inference at high lags since it estimates the distribution of the ACF for a series of length $l_b < T$, where b is the size of the blocks that are drawn from the observed sample of size T. However, since long memory is an asymptotic phenomenon, the slow rate of decay of the ACF of a long memory process can be detected only by looking at high lags. It is therefore necessary to estimate the distribution of the ACF for the entire sample and not just for the first l_b autocorrelations.

To overcome this limitation and make *optimal* inference at higher lags, we compare several alternative set ups to estimate the confidence bands around the ACF at all available lags to the benchmark represented by subsampling. Relative *optimality* is determined by minimizing a distance function with the subsampling confidence bands calculated for the first $l_b < T$ lags only.

More precisely, we use the following heuristic criterion to select *optimal* confidence bands for the ACF. Let:

$$\begin{aligned}
\mathbf{c}^s &\equiv \left[\mathbf{c}^s_{1,j} | \mathbf{c}^s_{2,j}\right] \\
&= \tanh \left[\begin{array}{c} (\widehat{z}_{T,1}, \ldots, \widehat{z}_{T,l_b})' \pm \\ \left(c_{\|\cdot\|,\widehat{z}^*_{b,1}} (1-\alpha/2) \times \widehat{\sigma}_{\widehat{z}^*_{b,1}}, \ldots, c_{\|\cdot\|,\widehat{z}^*_{b,l_b}} (1-\alpha/2) \times \widehat{\sigma}_{\widehat{z}^*_{b,l_b}} \right)' \end{array} \right],
\end{aligned}$$

be the subsampling confidence interval for the first l_b autocorrelations, where:

$$\widehat{z}_{T,j} \equiv \tanh^{-1}\left(\widehat{\rho}_j\right) = \frac{1}{2}\ln\frac{1+\widehat{\rho}_{T,j}}{1-\widehat{\rho}_{T,j}},$$

is the Fisher's z transform of the $j-th$ correlation ρ_j based on a sample of size T;

$$c_{\|\cdot\|,\widehat{z}^*_{b,j}}\left(1-\alpha/2\right),$$

is the $(1-\alpha/2)$ quantile of the distribution of the studentized $\widehat{z}^*_{b,j}$;

$$\widehat{\sigma}_{\widehat{z}^*_{b,j}} = \sqrt{\frac{1}{T-b}\sum_{i=1}^{T-b+1}\left(\widehat{z}^*_{b,j}-\overline{\widehat{z}}^*_{b,j}\right)^2},$$

is its standard deviation; $\mathbf{c}^s_{1,j}$ and $\mathbf{c}^s_{2,j}$ denote the upper and lower bounds of the interval, respectively.

The z transform has two main advantages over the autocorrelation coefficient, ρ. One is a symmetric distribution over the entire range of values $\rho \in (-1,1)$. Second, it ensures boundedness in the interval $[-1,1]$ of the confidence bands for the ACF. Additionally, Monte Carlo simulations show that the cumulative actual coverage probability of the subsampling confidence intervals for the first l_b autocorrelations improves substantially if estimated by using $z-$transform rather than the ACF directly. These results are available from the authors upon request.

Consider now the confidence bands for the first l autocorrelations, estimated by technique h:

$$\begin{aligned}\mathbf{c}^h &\equiv \left[\mathbf{c}^h_{1,j}|\mathbf{c}^h_{2,j}\right]\\ &= \tanh\left[\begin{array}{c}\left(\widehat{z}_{T,1},\ldots,\widehat{z}_{T,l}\right)'\pm\\ \left(c^{(h)}_{\|\cdot\|,\widehat{z}^*_1}\left(1-\alpha/2\right)\times\widehat{\sigma}^{(h)}_{\widehat{z}^*_1},\ldots,c^{(h)}_{\|\cdot\|,\widehat{z}^*_l}\left(1-\alpha/2\right)\times\widehat{\sigma}^{(h)}_{\widehat{z}^*_l}\right)'\end{array}\right],\end{aligned}$$

where $\widehat{z}_{T,l}$ is the lag l Fisher's z transform of the $l-$th sample autocorrelation $\widehat{\rho}_l$, estimated by using all the available observations, $\widehat{\sigma}^{(h)}_{\widehat{z}^*_l}$ is its standard deviations, and $c^{(h)}_{\|\cdot\|,\widehat{z}^*_l}(1-\alpha/2)$ is the $(1-\alpha/2)$ quantile of the distribution of the studentized z.

Let $\mathbf{e}^h = (\mathbf{c}^h - \mathbf{c}^s)$. The *optimal* confidence interval, $\widehat{\mathbf{c}}^{h_{OPT}}$, will be such that:

$$\widehat{\mathbf{c}}^{h_{OPT}} = \arg\min F(\mathbf{e}),$$

where

$$F(\mathbf{e}) = \sum_{i=1}^{2}\sum_{j=1}^{l}\left|c^h_{i,j} - c^s_{i,j}\right|. \tag{3}$$

In eq. (3), $c^h_{1,j}$ and $c^h_{2,j}$ are the upper and the lower bound of the confidence interval for the $j-th$ autocorrelation, estimated by using technique h.

4.1 Conditional $\beta-$Convergence and the ACF Framework

Analysis of the ACF of the detrended series of real (log) GDP per capita provides a framework for testing the conditional convergence hypothesis. Let $\rho_i(\tau)$ be the ACF of $y^d_{i,t} \equiv \left\{y_{i,t} - y^*_{i,t}\right\}$. The null hypothesis of no convergence - a unit root, in the standard set up given by (1) - implies a never decaying ACF while the alternative hypothesis of convergence to the steady state implies an exponential rate of decay of $\rho_i(\tau)$. The intermediate cases of slow convergence, denoted as fractional β-convergence by Michelacci and Zaffaroni (2000), imply an ACF that decays at a hyperbolic rate. Furthermore, an S-shaped ACF, as in Abadir and Talmain (2002), would imply that there is a slow tendency towards convergence (fractional convergence) with a persistent and non linear pattern.

More formally, building on Bernard and Durlauf (1995), we define conditional convergence as:

Definition 1 ***Conditional convergence in output.*** *The per capita GDP of country i, $y_{i,t}$, converges to its own long run steady state path, $y^*_{i,t}$, if the long term forecast of their* *difference is zero, at a given time t:*

$$\lim_{m\to\infty} E(y_{i,t+m} - y^*_{i,t+m} \mid I_t) = 0$$

which implies, in terms of the ACF of $\left\{y_{i,t} - y^*_{i,t}\right\}$, $\rho_i^d(\tau)$, *that there exists* τ^* *finite such that* $\rho_i^d(\tau) = 0$, $\forall \tau \geq \tau^*$.

To test for conditional convergence in per capita output, we detrend the series $\{y_{i,t}\}$ using a country-specific trend fitted by OLS, $y^*_{i,t}$; we define the residuals as $y^d_{i,t} \equiv \left\{y_{i,t} - y^*_{i,t}\right\}$ and estimate its ACF, $\rho_i^d(\tau)$, *via* eq. (2). By constructing confidence bands for $\rho_i^d(\tau)$ according to the procedure defined in the previous section we test, for country i:

$$H_0 : \rho_i^d(\tau) = 0 \quad vs. \quad H_A : \rho_i^d(\tau) \neq 0, \forall \tau \geq \tau^*.$$

The null hypothesis of conditional convergence cannot be rejected if there exists a finite value of τ, τ^*, such that the confidence bands for $\rho_i^d(\tau)$ contain zero for any $\tau \geq \tau^*$.

The standard definition of conditional convergence implies that there exists a value τ^* such that $\rho_i^d(\tau) = 0$ for any $\tau \geq \tau^*$, where τ^* is the *first* lag at which the ACF goes to zero. In other words, tests for convergence based on a linear autoregressive framework imply, in terms of the ACF, a monotonic decay towards zero, either exponential or hyperbolic. By contrast, Definition (1) allows for the possibility of non-monotonic decay of $\rho_i^d(\tau)$, since it imposes no constraint on the pattern that convergence processes must follow. Our definition therefore encompasses the standard definition of convergence as a special case, but allows for convergence patterns that standard procedures would fail to detect.

4.2 Transitional Dynamics in the Log-linearized Solow Model

The ACF-based framework developed above allows an assessment of the empirical validity of the neoclassical model, in its log-linearized version. Consider an economy where rates of investment, technological progress, population growth, capital depreciation and the elasticity of output with respect to capital are exogenous and constant over the observed time period. Constant labour force participation is assumed, since available data is real per capita output rather than the theoretically preferable real output per unit of labour.

Let the production function be Cobb-Douglas, with labor-augmenting technological progress

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}, \tag{4}$$

where Y_t, K_t, L_t, A_t, are output, capital, labor and A is the level of technology, all measured at time t, with $0 < \alpha < 1$. A and L are assumed to grow exogenously at rate g and n, respectively. A constant fraction of output, s, is invested. Defining $\hat{k} = K/AL$ and $\hat{y} = Y/AL$ as the stock of capital and output per effective unit of labor, the evolution of $\hat{k}$ is given by:

$$\frac{d\hat{k}_t}{dt} = s\hat{k}_t^{\alpha} - (n + \delta + g)\hat{k}_t, \tag{5}$$

where δ is the depreciation rate. Under the hypothesis of diminishing returns to capital, equation (5) implies that $\hat{k}_t$ converges to its steady state level $\hat{k}_t^*$:

$$\hat{k}^* = \left(\frac{s}{n + \delta + g}\right)^{1/(1-\alpha)}. \tag{6}$$

Following Mankiw, Romer and Weil (1992), consider a log-linear approximation of equation (6) around the steady state:

$$\frac{d[\ln(\hat{y}_t)]}{dt} = -\beta[\ln(\hat{y}_t) - \ln(\hat{y}_t^*)], \tag{7}$$

where $\hat{y}_t^* = (\hat{k}_t^*)^{\alpha}$ and $\beta = (1 - \alpha)(n + \delta + g)$.

Equation (7) does not differ qualitatively from that implied by the neoclassical model augmented with human capital. In this model, the economy would still converge to its equilibrium at an exponential rate, but the speed will be slower: the rate of convergence is

$\beta' = (1 - \lambda - \alpha)(n + \delta + g)$, where λ measures the elasticity of output to human capital. Equation (7) would, however, still be the framework used to test for convergence processes.

By discretizing (7) and making use of Jones (1995) invariance property, which states that the trend of output per capita for OECD economies is smooth over time, so that $y_{it}^* = g_i t + y_{i,t_o}$, where y_{i,t_o} is the initial condition, one obtains:

$$y_{i,t} - y_{i,t-1} = g_i + \beta_i y_{i,t-1}^* - \beta_i y_{i,t-1}. \tag{8}$$

Subtracting from both sides $y_{i,t}^*$ and rearranging terms, (8) can be rewritten as

$$\begin{aligned} y_{i,t} - y_{i,t}^* &= g_i + \beta_i y_{i,t-1}^* + (1 - \beta_i)\, y_{i,t-1} - y_{i,t}^* \\ &= g_i + (1 - \beta_i)\, y_{i,t-1} + \beta_i y_{i,t-1}^* - \left(g_i - y_{i,t-1}^*\right), \end{aligned}$$

which gives

$$y_{i,t} - y_{i,t}^* = (1 - \beta_i)\left(y_{i,t-1} - y_{i,t-1}^*\right),$$

that is, an AR(1) process whose theoretical ACF is:

$$\rho_i^{AR1}(\tau) = (1 - \beta_i)^{\tau}. \tag{9}$$

We then estimate by Nonlinear Least Squares:

$$\rho_i^d(\tau) = \rho_i^{AR1}(\tau) + u(\tau), \qquad \tau = 1, 2, \ldots, k \tag{10}$$

and test whether the fitted ACF implied by the neoclassical model, $\widehat{\rho}_i^{AR1}(\tau)$, and the sample ACF, $\rho_i^d(\tau)$, are not different:

$$H_0 : \rho_i^{AR1}(\tau) = \rho_i^d(\tau), \quad \forall \tau \qquad vs \quad H_A : \rho_i^{AR1}(\tau) \neq \rho_i^d(\tau), \quad \text{for at least one } \tau.$$

If the null hypothesis is rejected, we conclude that the model is unable to replicate the convergence process of an economy to its equilibrium level.

4.3 Convergence in International Output

Convergence among OECD economies remains controversial in the empirical growth literature. Some researchers suggest that these economies behave like a club (see, among others, Barro and Sala-i-Martin, 1992; Bianchi, 1997; Quah, 1997; Michelacci and Zaffaroni, 2000). Alternatively, it is argued that there exist multiple equilibria among OECD economies (Durlauf, 1993; Bernard and Durlauf, 1995; Durlauf and Johnson, 1995; Canova, 2004).

To help resolve this debate, we use the ACF framework to test whether economies tend to the same steady state path. A common trend, estimated by GLS, is used to detrend real GDP per capita. Using the level of real GDP per capita of a benchmark country (as in Bernard and Durlauf, 1995) or a common trend (as in Michelacci and Zaffaroni 2000) and then examining the properties of the deviations from this trend is a standard procedure to test for unconditional (or pairwise, or club) convergence in a time series environment.

More formally, we define unconditional club convergence as:

Definition 2 (Unconditional convergence in output.) *The per capita GDP of countries $i = 1, ..., n$ converge to a common long run equilibrium path, $\bar{y}_{i,t}$, if the long term forecast of each country's per capita output is equal to the common equilibrium path, at a given time t:*

$$\lim_{m \to \infty} E(y_{i,t+m} - \bar{y}_{t+m} \,|I_t) = 0,\ \forall i,$$

which which implies, in terms of the ACF of $\{y_{i,t} - \bar{y}_t\}$, $\bar{\rho}_i(\tau)$, that there exists τ^* finite such that $\bar{\rho}_i(\tau) = 0$, $\forall \tau \geq \tau^*$.

Let $\{y_{i,t}\}$ be the (log of) real output per capita of country i, with $i = 1, ..., N$, and $\bar{y}_t = gt + y_0$ the common trend, estimated using NT realizations of GDP per capita. Let $\bar{\rho}_i(\tau)$ be the ACF of the series $\{y_{i,t} - \bar{y}_t\}$.

Testing unconditional convergence amounts to estimating the ACF of $\{y_{i,t} - \bar{y}_t\}$ for all $i = 1, ...N$, and constructing confidence bands around them: if there exists a finite value τ^* such that the confidence bands contain zero for all $\tau \geq \tau^*$ and for all $i = 1, \ldots, N$, then unconditional club convergence exists.

5 Discussion of Results

Figure 2 and 3 show our empirical results for conditional convergence. Results for the U.S., U.K. and Italy represent three different convergence processes and are plotted in Figure 2. Processes for other economies are plotted in Figure 3.

In all cases, the 95% confidence bands we plot have been estimated by stationary bootstrap (Politis and Romano, 1994), which is, in terms of minimization of the distance function (3), superior to the jackknife (Quenouille, 1949), to the moving block bootstrap (Kunsch, 1989; Hall, Horowitz and Jing, 1995), and to the sieve bootstrap (Buhlmann, 1997).

<Insert Figure 2 here>

<Insert Figure 3 here>

These results deserve further comments. First, conditional convergence cannot be altogether rejected. It holds for the U.S., Austria, Belgium, France, Japan, Netherlands, Norway and U.K. Other economies show no tendency to converge to the steady state. Second, there are, however, striking differences among economies that show convergence to their own steady state. While the ACF of U.S. detrended real output per capita declines to zero monotonically and at an exponential rate, with $\tau^* = 4$, Austria, Belgium, France, Japan, Netherlands, Norway and U.K. show a much slower, and less clear cut, convergence pattern towards the steady state, with τ^* varying from 46 and 70 years. Third, the monotonic decay in the ACF of U.S. detrended output suggests that the hypothesis of conditional convergence, in its standard and strongest formulation of exponential β-convergence, cannot be rejected in this case, but in the other cases, the ACF becomes insignificantly different from zero at a very slow rate and, more interestingly, the pattern is non-monotonic. This implies that the standard definition of β-convergence can be rejected for these countries, even in the more general form of fractional β-convergence, since this can account for the slow rate of convergence but not for the non-monotonicity of the transitional dynamics. According to the ACF-based definition given in this paper, β-convergence cannot be rejected. Fourth, we find that Australia, Canada, Denmark, Finland, Germany, Italy, and Sweden show no evidence of conditional convergence for the period examined. For example, the ACF of detrended output for Italy does not revert to zero, implying that the null hypothesis of conditional convergence must be rejected even in the general formulation of Definition 1.

Taken as a whole, two main conclusions follow from our results. One is that, in sharp contrast with much of the previous literature on convergence, conditional β-convergence is not a common feature across OECD countries: some economies tend to their steady state position, but others do not. Second, in line with Lee, Pesaran and Smith (1997), we find no evidence of the homogenous, exponential 2% speed of convergence for all the economies, suggested by Barro and Sala-i-Martin (1992).

Figure 4 shows results for the log-linearized Solow model. Results are reported for three representative cases only: U.S., U.K. and Italy.

<Insert Figure 4 here>

The log-linearized neoclassical model can replicate the pattern of the empirical ACF only for the U.S. economy. For the U.K. and Italy, the fitted ACF is not contained within the 95% confidence bands constructed for the empirical ACF. Accordingly, the null hypothesis that the fitted ACF and the empirical ACF are not statistically different must be rejected. The same holds for all the remaining cases (see Caggiano and Leonida, 2006, for results on all the 15 economies). The linearized Solow model can account for exponential conditional convergence, for the U.S., but not for convergence patterns in all other OECD economies. These results call into question the usefulness of the log-linearization of the Solow model, introduced by Mankiw, Romer and Weil (1992), as a general empirical framework to analyze growth and convergence and are consistent with the remarks recently made by Mathunjwa and Temple (2006).

Figure 5 and 6 give results for unconditional, or club, convergence. Findings for the U.S., U.K. and Italy are shown in Figure 5, and results for all other economies in Figure 6.

<Insert Figure 5 here>

<Insert Figure 6 here>

Our findings suggest that OECD economies do not converge to a unique equilibrium path. Furthermore, the dynamic pattern is heterogenous across countries. Although some economies appear slowly to converge to a common steady state path, their dynamics show remarkable differences. The ACFs of the U.S. and the U.K. always remain positive, while the ACF of Italy shows persistent and significant fluctuations around zero. Moreover, when there is a transition to the common trend, the pattern is neither of the exponential nor of the fractional type usually considered in the literature.

Even for the U.S., there is no evidence of convergence towards a common OECD equilibrium. This may signal the existence of important country-specific structural characteristics.

If so, the neoclassical assumption of an homogenous aggregate production function may not be innocuous when testing for cross-country convergence. Instead, as argued in the literature on multiple regimes, the explanatory power of the Solow model may be enhanced by allowing for heterogenous aggregate production functions (Durlauf and Johnson, 1995) and by including differences in technology (Bernard and Jones, 1996).

A thorough investigation of the impact of country-specific factors on transitional dynamics is beyond the scope of this paper. However, we use a simple correlation analysis to get preliminary evidence. We estimate the correlation between persistence in transitional dynamics and a (non-exhaustive) number of variables that are either assumed constant or are not included in the Solow growth model: among them, the variability of saving rates, government size and openness. We also include the initial level of income among the regressors.[1]

We find that the level of persistence is positively correlated with saving rate variability, government size and openness (the estimated coefficients are 0.27, 0.19, 0.22, respectively, and are all significant at a 1% level), and negatively correlated with the initial level of wealth (the estimated coefficient is -0.13 and is significant at a 10% level). Positive correlation between variability of saving rates and convergence patterns suggests that the further an economy is from Solow's assumption of a constant saving rate, the greater the importance of the transitional dynamics relative to its steady state behaviour. Variable saving rates can account for the presence of non-convexities in production, as suggested by Galor (1996), and this in turn may explain nonstandard and persistent transitional patterns. The positive sign associated with openness is indicative of a slower transition in economies which are particularly exposed

[1] Data are taken from the Penn World Table. The variability of the saving rate is calculated as the variance over 5 years (that is the variance calculated for the period 1950-1955, 1956-1960, etc.). Persistence in the transitional dynamics of country i is measured as the fifth-lag autocorrelation calculated for different samples: 1900-2000, 1900-1995, 1900-1990 and so on.

to external shocks: the less an economy is open, the more its convergence pattern will replicate that implied by the Solow model. This is the case of the U.S., the economy with the lowest openness index in our sample and the only economy to follow a transitional pattern close to that implied by the Solow model. Finally, the negative relationship associated with the initial level of income can be interpreted as evidence that initial conditions have an impact on the importance of the transitional dynamics relative to the steady-state behaviour. This supports the conclusions of Mathunjwa and Temple (2006) that the effect of initial income is not homogenous across countries and is related to their claim that the validity of log-linearization crucially depends on the distance from the equilibrium path. These findings raise interesting questions that cannot be addressed in this paper but which we propose to examine in future research.

6 Concluding Remarks

We use an approach based on the ACF of detrended output per capita to test for conditional and unconditional convergence in a group of 15 OECD countries and to determine whether the empirical implications of the neoclassical model of growth, in its standard loglinearized version, are consistent with the observed transitional dynamics.

The contribution of this paper to the literature on growth and convergence processes can be summarized as five main points. First, our procedure yields results distinct from most alternative time series tests for convergence. Difference is apparent, in particular, compared to an approach based on unit root testing, mainly because in it the sum of the null and of the alternative hypotheses does not include all the possible underlying DGPs for the series examined. Imposing a linear constraint on the DGP of detrended output and forcing the data to be either $I(1)$ or $I(0)$ is likely to be misleading when examining convergence properties of different economies. Allowing for $I(d)$ processes, in the spirit of Michelacci and Zaffaroni (2000), is a first step towards a correct understanding of transitional dynamics. Furthermore, allowance for nonlinear dynamics contributes to a better understanding of the data.

Second, unlike much of the previous literature on growth convergence, our findings indicate that conditional convergence does not occur, even across OECD economies. Indeed, a number of these economies show no clear tendency to revert to their own steady state position if, as usual in the literature, equilibrium is approximated by a log-linear trend. Observed differences in transitional dynamics may be due to the unaccounted presence of idiosyncratic factors, such as time-varying saving rates, government size, openness and the initial level of income.

Third, even when convergence occurs, we show that the transitional process, the U.S. apart, is not of the type (exponential or fractional) considered in the previous literature.

Fourth, like Bernard and Durlauf (1995) but unlike much of the literature, we reject the hypothesis of unconditional (or club) convergence.

Finally, the paper shows that the loglinearized version of the neoclassical growth model, standard and universally employed in the empirical growth literature, cannot replicate the actual convergence processes of 14 of the 15 OECD economies we examine. It captures only U.S. transitional dynamics. Whether this is due to log-linearization itself or to some assumptions of the model remains, however, a question for future research.

References

[1] Abadir, K. M. and G. Talmain (2002), "Aggregation, persistence and volatility in a macro model", *Review of Economic Studies*, **69**, 749-779.

[2] Abadir, K. M. and A. M. R. Taylor (1999), "On the definitions of (co)-integration", *Journal of Time Series Analysis*, **20**, 129-137.

[3] Abramowitz, M. (1986), "Catching up, Forging Ahead and Falling Behind", *Journal of Economic History*, **46**, 385-406.

[4] Barro, R. J. and X. Sala-i-Martin (1992), "Convergence", *Journal of Political Economy,* **100**, 223-251.

[5] Baumol, W. J. (1986), "Productivity Growth, Convergence and Welfare. What Long-Run Data Show", *American Economic Review*, **76**, 1072-1085.

[6] Bernard, A. B. and S. N. Durlauf (1995), "Convergence in International Output", *Journal of Applied Econometrics*, **10**, 97–108.

[7] Bernard, A. B. and S. N. Durlauf (1996), "Interpreting tests of the Convergence Hypothesis", *Journal of Econometrics*, **71**, 161–173.

[8] Bernard, A. B. and C.I. Jones (1996), "Technology and Convergence", *The Economic Journal*, **106**, 1037–1044.

[9] Bianchi, M. (1997), "Testing for convergence: evidence from nonparametric multimodality test", *Journal of Applied Econometrics*, **12**, 393-409.

[10] Box, G. E.P. and G. M. Jenkins (1976) *Time Series Analysis: Forecasting and Control*, revised ed., San Francisco: Holden-Day.

[11] Buhlmann, P. (1997), "Sieve bootstrap for time series", *Bernoulli*, **3**, 123-148.

[12] Caggiano, G. and L. Leonida (2006), "A note on the empirics of the neoclassical growth model", *Economics Letters*, forthcoming.

[13] Canova, F. (2004), "Testing for Convergence Clubs in Income Per Capita: A Predictive Density Approach", *International Economic Review*, **45**, 49-77.

[14] Diebold, F. X., and G. D. Rudebusch (1989), "Long memory and persistence in aggregate output", *Journal of Monetary Economics*, **24**, 189-209.

[15] Durlauf, S. N.(1993), "Nonergodic Economic Growth", *Review of Economic Studies,* **60**, 349–366.

[16] Durlauf, S. N. and P. A. Johnson (1995), "Multiple Regimes and Cross-Country Growth Behavior", *Journal of Applied Econometrics,* **10**, 365–384.

[17] Durlauf, S. N. and D. Quah (1999), "The New Empirics of Economic Growth", in *Handbook of Macroeconomics,* J. Taylor and M. Woodford, eds., Amsterdam: North Holland.

[18] Galor, O. (1996), "Convergence? Inferences from Theoretical Models", *The Economic Journal,* **106**, 1056–1069.

[19] Gil-Alãna, L. A., and P. M. Robinson (1997), "Testing of unit root and other nonstationary hypotheses in macroeconomic time series", *Journal of Econometrics*, **80**, 241-68.

[20] Hall, P., Horowitz, J. and B.-Y. Jing (1995), "On blocking rules for the bootstrap with dependent data", *Biometrika*, **82**, 561-574.

[21] Islam, N. (2003), "What Have We Learnt from the Convergence Debate?", *Journal of Economic Surveys*, **17**, 309-362.

[22] Jones, C. I. (1995), "Time Series Tests of Endogenous Growth Models", *Quarterly Journal of Economics*, **110**, 495-525.

[23] Kunsch, H. R. (1989), "The jackknife and the bootstrap for general stationary observations", *Annals of Statistics*, **17**, 1217-1241.

[24] Lee, K., Pesaran, M. H. and R. Smith (1997), "Growth and Convergence in a Multy-Country Empirical Stochastic Solow Model", *Journal of Applied Econometrics*, **12**, 357-392.

[25] Levine, R. and D. Renelt, (1992), "A Sensitivity Analysis of Cross Country Growth Regressions", *American Economic Review*, **82**, 942-963.

[26] Mankiw, N., Romer, D. and D. Weil (1992), "A Contribution to the Empirics of Economic Growth", *Quarterly Journal of Economics*, **107**, 407-437.

[27] Mathunjwa, J. S. and J. Temple (2006), "Convergence behaviour in exogenous growth models", Discussion Paper No. 06/590, Department of Economics, University of Bristol.

[28] Michelacci, C. and P. Zaffaroni (2000), "(Fractional) beta convergence", *Journal of Monetary Economics*, **45**, 129-153.

[29] Politis, D. N. and J. P. Romano (1994), "The Stationary Bootstrap", *Journal of the American Statistical Association*, **89**, 1303-1313.

[30] Politis, D. N., Romano, J. P. and M. Wolf (2004), "Inference for Autocorrelations in the Possible Presence of a Unit Root", *Journal of Time Series Analysis,* **25***,* 251-263.

[31] Quah, D. T. (1996), "Empirics for Economic Growth and Convergence", *European Economic Review*, **40**, 1353-1375.

[32] Quah, D. T. (1997), "Empirics for Growth and Distribution: Stratification, Polarization and Convergence Clubs", *Journal of Economic Growth,* **2**, 27-59.

[33] Quenouille, M. H. (1949), "Approximate tests of correlation in time-series", *Journal of the Royal Statistical Society Series B*, **11**, 68-84.

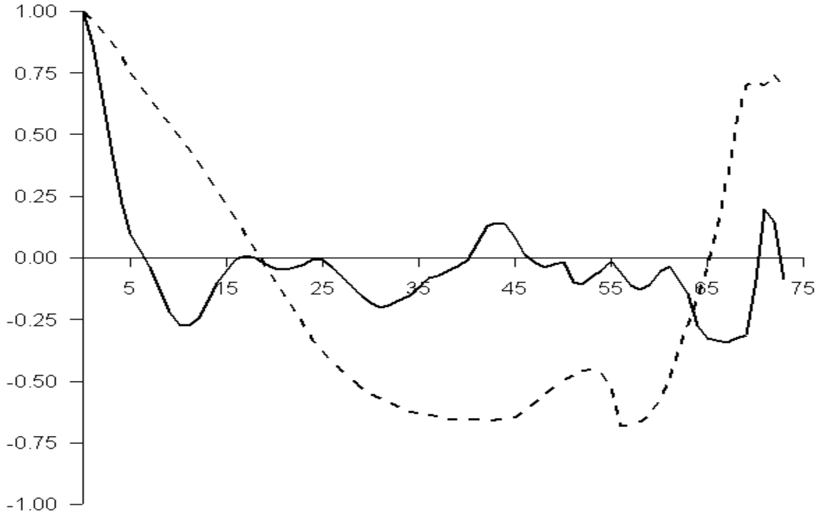
[34] Silverberg, G. and B. Verspagen (2000), "A note on Michelacci and Zaffaroni, long memory, and time series of economic growth", ECIS Working Papers 00.17, Eindhoven Centre for Innovation Studies, Eindhoven University of Technology.

[35] Solow, R. M. (1956) "A Contribution to the Theory of Economic Growth", *Quarterly Journal of Economics,* **70**: 65-94.

Figure 1
Empirical ACF

Panel A reports the ACF function estimated for the U.S (solid line) and Japan (dashed line). Panel B reports ACFs estimated for all countries.

Panel A: U.S. (full line) and Japan (dashed line)



Panel B: All economies

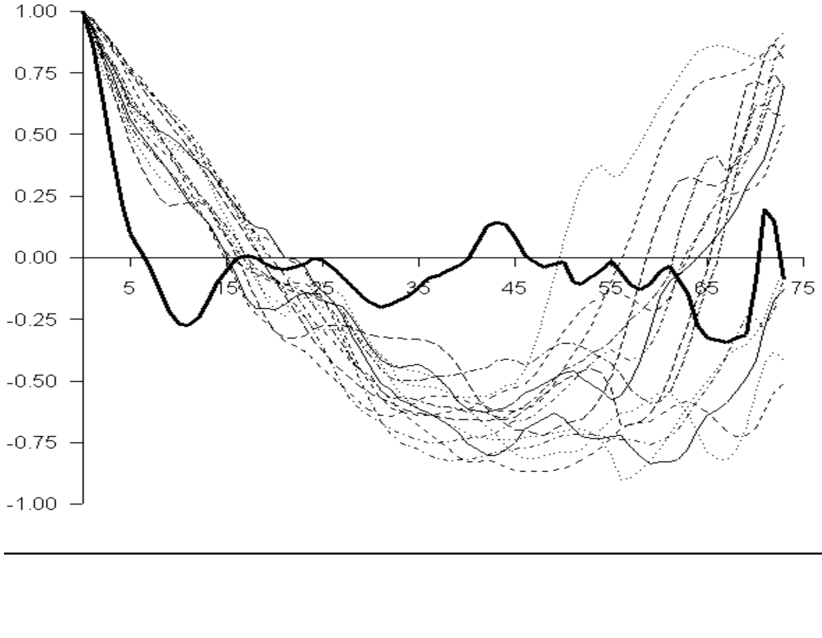


Figure 2

Conditional Convergence (Selected Economies)

The solid lines represent the autocorrelation functions. The dashed lines are the confidence band to test the null hypothesis that the ACF is statistically insignificant at 95% c.l.

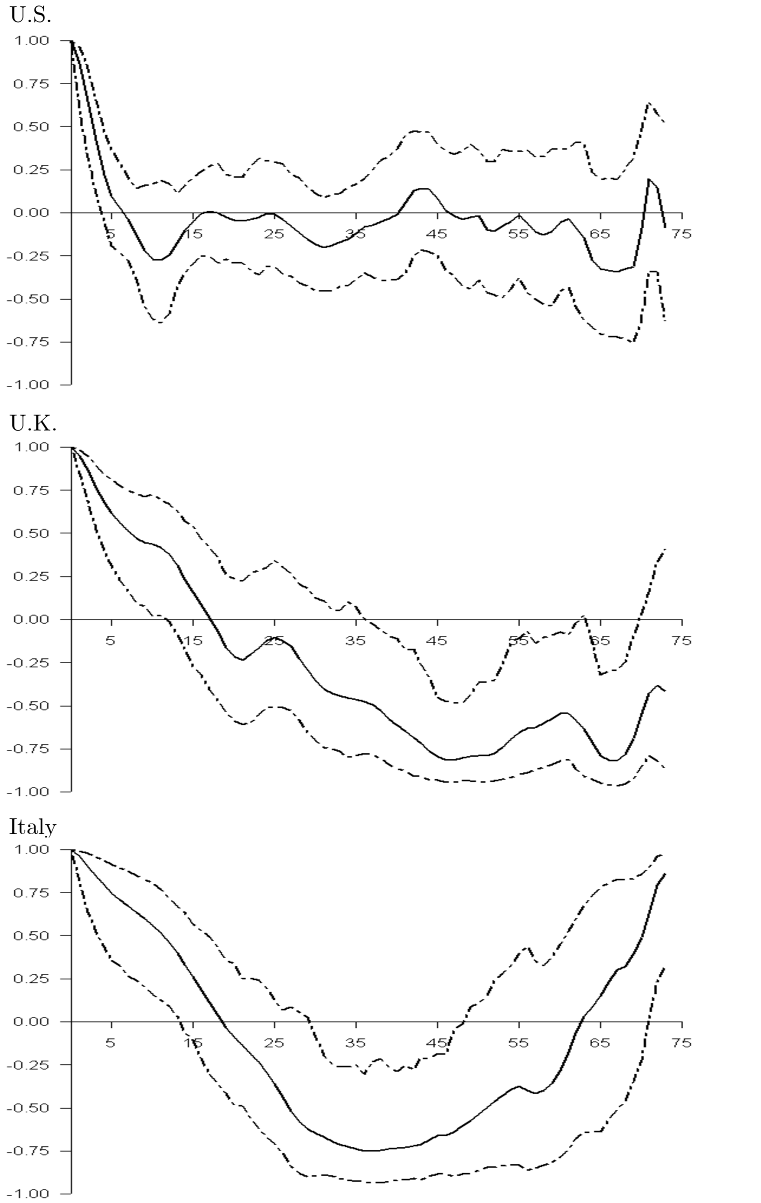


Figure 3

Conditional Convergence (All Economies)

The solid line represents the autocorrelation function. The dashed lines are the confidence band to test the null hypothesis that the ACF is statistically insignificant at 95% c.l.

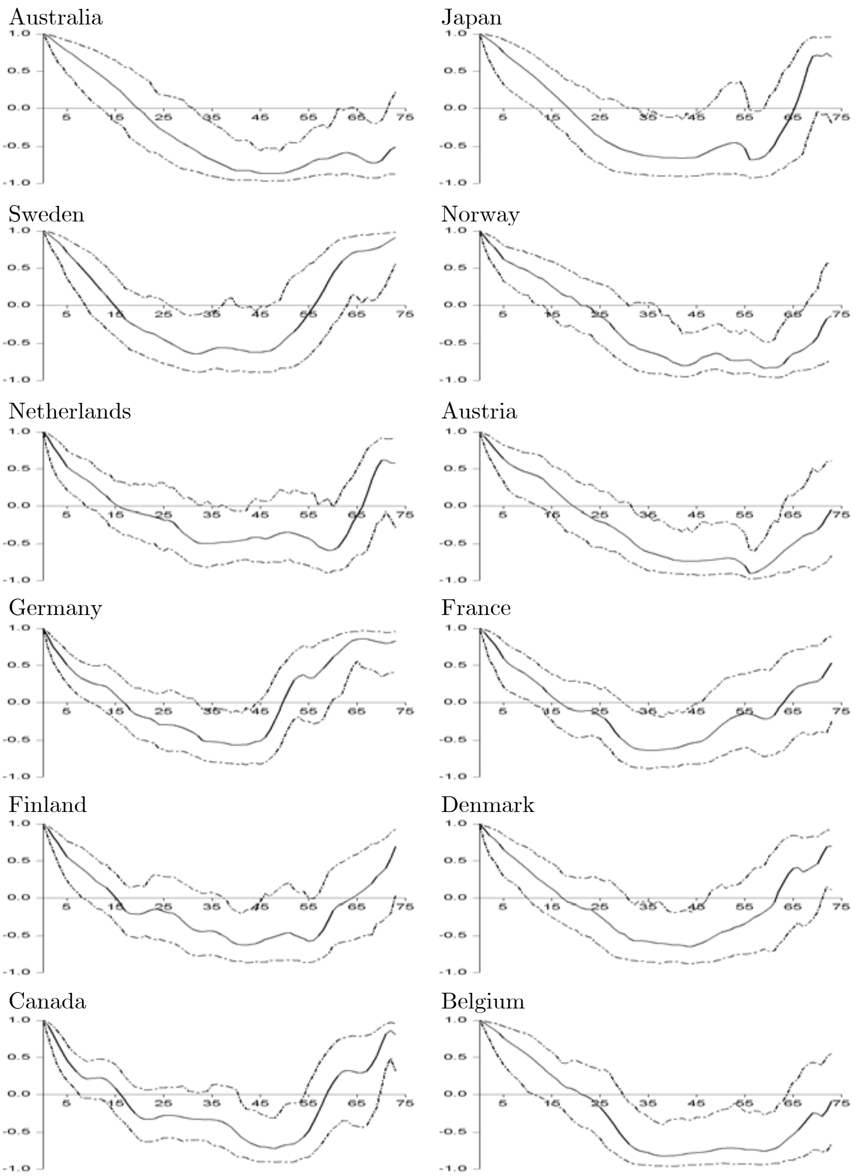


Figure 4

Fitted and Empirical ACF (Selected Economies)

The solid lines represent the autocorrelation functions. The dashed lines are the fitted ACFs. The dot-dashed lines are the confidence bands to test the null hypothesis that the ACF is statistically insignificant at 95% c.l.

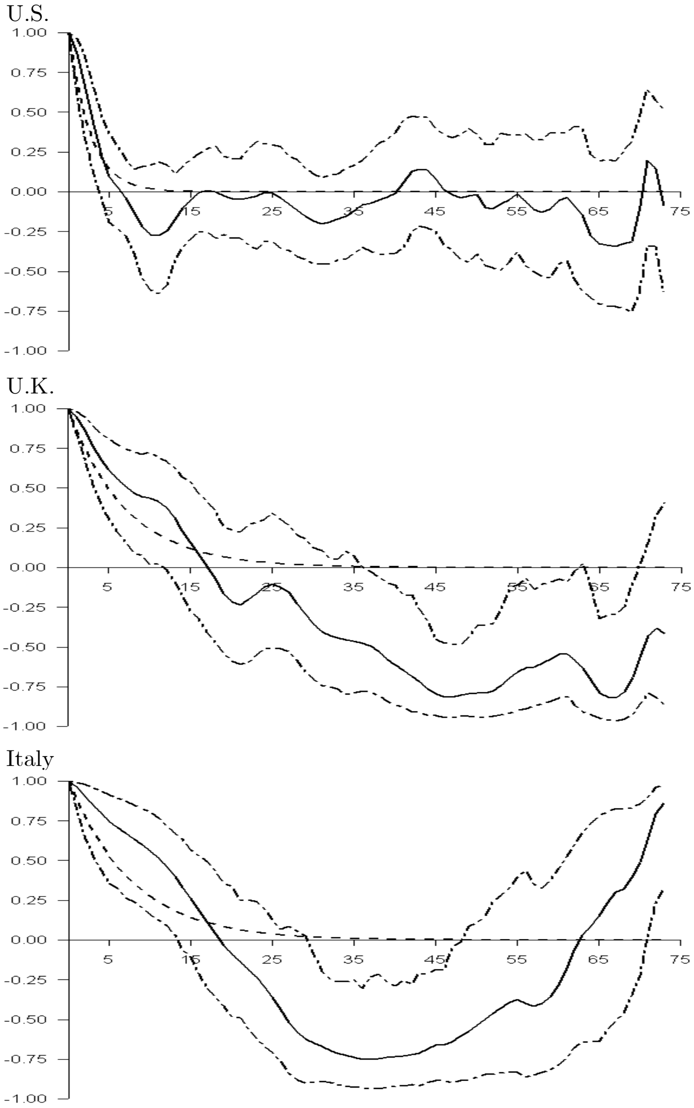


Figure 5
Unconditional Convergence (Selected Economies)

The solid lines represent the autocorrelation functions. The dashed lines are the confidence band to test the null hypothesis that the ACF is statistically insignificant at 95% c.l.

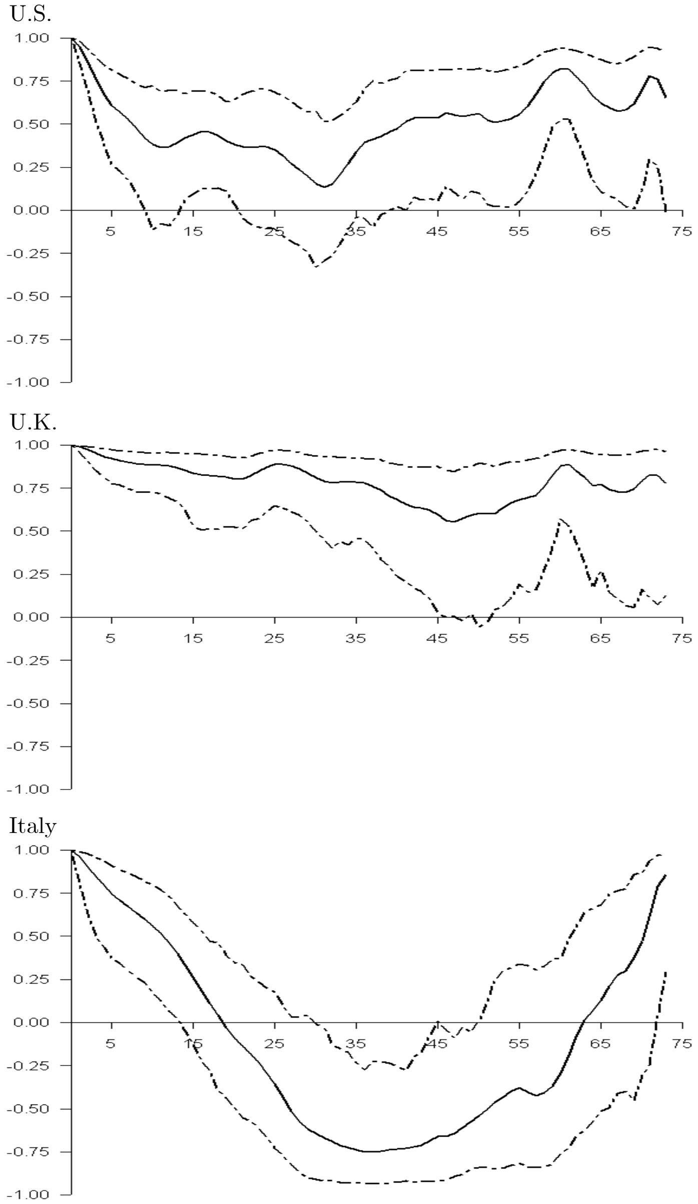


Figure 6
Unconditional Convergence

The solid line represents the autocorrelation function. The dashed lines are the confidence band to test the null hypothesis that the ACF is statistically insignificant at 95% c.l.