A note on testing for tax-smoothing in general equilibrium

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Abstract
Barro's original partial equilibrium tax-smoothing model has generated a tremendous amount of empirical interest over the last several decades. However, to date, there has been no formal empirical testing of the more recent general equilibrium renditions of this model. Therefore, the purpose of this paper is to construct, and directly test, a general equilibrium model of optimal growth and endogenous fiscal policy in which policymakers find it optimal to keep the tax rate constant over time. In contrast to most of the evidence from partial equilibrium models, we find that data from 26 OECD economies uniformly reject the tax-smoothing hypothesis over the period 1960-1996.

Keywords: Fiscal policy and private agents, Optimal taxation, Growth

JEL classification: H3, H21

1. Introduction

One of the most popular models for the analysis of endogenous fiscal policy in an intertemporal framework is Barro's [1979] tax-smoothing model. The original model is cast in a partial equilibrium context and prescribes that the tax rate should be smoothed over time.[1] The basic premise is that tax policy is distorting and therefore the optimizing fiscal authorities allocate this policy over time to minimize its negative effects. More recently, general equilibrium versions of the tax-smoothing model have been employed in the theoretical literature on optimal growth and endogenous fiscal policy (see, e.g. Chari *et al*. [1994] and for simpler models Barro and Sala-i-Martin [1992], Benhabib and Velasco [1996] and Devereux and Wen [1998]).

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[1] Barro [1979] uses an *ad hoc* quadratic loss function that depends only on current tax collections. Also, the behavior of the private sector is taken as given, which can be interpreted as a strong form of Ricardian Equivalence. This model implies that the tax rate should change only if there are unanticipated shocks, i.e. the tax rate should follow a random walk independently of the stochastic processes of the underlying shocks.

While the partial equilibrium tax-smoothing model has generated a tremendous amount of empirical interest over the last several decades (see, e.g. Serletis and Schorn [1999] and the references cited therein)[2], there has been, surprisingly, no testing of the general equilibrium renditions.[3] Accordingly, the purpose of this paper is to construct, and formally test, a general equilibrium model of optimal growth and endogenous fiscal policy in which policymakers find it optimal to keep the tax rate constant over time.

The paper is organized as follows. In Section 2, we obtain a closed-form analytical solution which consists of behavioral relations for private consumption-to-, private capital-to- and government expenditure-to-output ratios. This general equilibrium is Markov-perfect, and hence optimal policy is time consistent. The closed-form solution enables us, in Section 3, to *directly* test the cross-equation restrictions implied by the interaction between optimizing private agents and fiscal authorities.[4] Our empirical testing is conducted using data from 26 OECD economies over the period 1960-1996. We find that the data resoundingly reject the empirical validity of the model's over-identifying restriction and hence the tax-smoothing hypothesis. Hence, it appears that testing based on partial equilibrium models has over-favored the tax-smoothing hypothesis of policymaking. This is consistent with the finding of Chari et al. [1994] for the U.S.. Finally in Section 4 we discuss our conclusions and related research.

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[2] Serletis and Schorn [1999] find support for the tax-smoothing hypothesis using data from Canada, France, the U.K. and the U.S.. Their results are consistent with previous evidence for tax-smoothing, at least at federal levels of government.

[3] A notable exception in the Real Business Cycles (RBC) tradition is Chari *et al*. [1994].

[4] RBC models have also incorporated fiscal policy (see, e.g. Christiano and Eichenbaum [1992], Baxter and King [1993], McGrattan [1994] and Ambler and Paquet [1996]). For tests of RBC models with tax-smoothing, see Chari *et al*. [1994] and Stokey and Rebelo [1995]. However, most of these models are "tested" with the use of calibration techniques following the RBC tradition. In contrast, here we obtain closed-form analytical solutions and hence can use formal econometric techniques to directly test the theoretical cross-equation

2. Growth and fiscal policy: the tax-smoothing model

Consider a closed economy consisting of a representative private agent and a government sector. The private agent chooses time-paths of consumption and capital to solve an intertemporal optimization problem. In doing so, he acts competitively by taking prices and economic policy as given. The government finances the provision of public consumption services[5] by taxing the private agent's income. Thus, taxes are distortionary. The government is benevolent and acts as a Stackelberg leader *vis-à-vis* the private sector. Time is discrete, the time-horizon is infinite and, for simplicity, there is no uncertainty.

Here we solve for Markov strategies, i.e. players' optimal strategies are functions of the current value of the relevant state variables. That is, we solve for Markov-perfect general equilibria which are sub-game perfect and hence time-consistent. This is important because when taxes are distorting, optimal policy is inherently time-inconsistent.

2.1 Private sector

The representative agent maximizes intertemporal utility:

$$\sum_{t=0}^{\infty} b^{t}[\log c_t + d \log g_t]$$

(1)

where c_t is private consumption at t, g_t is public consumption services at t, $0 < b < 1$ is the discount rate and $d \geq 0$ is the weight given to public consumption services relative to private consumption. For simplicity, we use a logarithmic and additively separable utility function.

The flow budget constraint of the representative agent is:

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restrictions of a general equilibrium model with endogenous fiscal policy. Christiano and Eichenbaum [1992] also use formal econometric techniques but in their model policy is exogenous.

[5] Our results do not change if we also include public production services that provide production externalities to private firms and so generate long-run growth. For details, see footnote 12 below.

$$k_{t+1} + c_t = (1 - q_t)Ak_t^a$$

(2)

where k_{t+1} is the end-of-period capital stock, k_t is the beginning-of-period capital stock, $y_t = Ak_t^a$ is current output produced via a Cobb-Douglas technology (where $A > 0$ and $0 < a < 1$), and $0 \quad q_t \quad 1$ is the income tax rate. The initial capital stock is given. Note that (2) assumes full capital depreciation within a single period.

We formulate the problem as a dynamic programming one. From the competitive private agent's viewpoint, the state at time t can be summarized by the predetermined capital stock, k_t, and the current tax rate, q_t.[6] Then, if $U(k_t;q_t)$ denotes the value function of the private agent at t, by using standard dynamic programming techniques (see, e.g. Sargent [1987, chapter 1] and Stokey and Lucas [1989, chapter 4]), it is easy to show that, for Markov tax strategies, consumption, c_t, and the end-of-period capital stock, k_{t+1}, are:[7]

$$c_t = (1 - ab)(1 - q_t)Ak_t^a \tag{3}$$

$$k_{t+1} = ab(1 - q_t)Ak_t^a . \tag{4}$$

2.2 The government sector and competitive equilibrium (given policy)

At each t, the government finances g_t by taxing the private agent's income.[8] Thus,

$$g_t = q_t Ak_t^a .$$

(5)

[6] Since the private agent acts competitively, he takes q_t and g_t as given.

[7] For details, see Malley and Philippopoulos [1999]. The fact that the competitive private agent's decisions are obtained as the policy solutions to a dynamic programming problem, in combination with the requirement that fiscal policy variables are Markov, makes the competitive equilibrium a recursive one. In other words, allocations and prices are functions of the current value of the relevant state variables. In turn, the problem of the government becomes also recursive and its strategies are Markov (see Kollintzas *et al*. [1999]).

[8] That is, there is no public debt. This is for simplicity because we want to obtain closed-form solutions. In any case, this assumption is not unusual in the relevant literature (see e.g. Barro and Sala-i-Martin [1992], Baxter and King [1993], McGrattan [1994], Ambler and Paquet [1996] and Benhabib and Velasco [1996]). Adding public debt would not change our main results (it would just make them "smoother", see Chari *et al*. [1994]). Also, by omitting public debt, we avoid well-known data measurement problems.

Equations (3), (4) and (5) give a competitive equilibrium in c_t, k_{t+1} and one of the two policy instruments, q_t and g_t.[9] In this equilibrium: (i) private agents maximize their intertemporal utility; (ii) all markets clear; and (iii) the government's budget constraint is satisfied. Looking ahead at the empirical work below, it is convenient to express this equilibrium in terms of g_t. Thus, solving (5) for q_t and substituting into (3) and (4), we obtain:

$$c_t = (1 - ab)(Ak_t^a - g_t) \tag{6}$$

$$k_{t+1} = ab(Ak_t^a - g_t) \,. \tag{7}$$

Equations (6) and (7) give a Competitive Equilibrium for any feasible level of g_t.[10]

2.3 Endogenous fiscal policy and general equilibrium

We assume that the government chooses its policy to maximize the private agent's lifetime utility. By acting as a Stackelberg leader *vis-à-vis* the private agents, the government chooses a path of g_t to maximize (1) subject to (6) and (7). The resulting Markov strategy for g_t, in combination with (6) and (7), will give a Markov-perfect general equilibrium.

From the government's viewpoint, the state at t is the predetermined capital stock, k_t. Then, if we denote by $V(k_t)$ the value function of the government at t, and use standard dynamic programming techniques, it is straightforward to show that it is optimal to keep the government expenditures-to-output ratio $\frac{g_t}{y_t}$, or equivalently via (5) the income tax rate q_t, constant over time. In particular, the government's Markov strategy is:

$$0 < q_t = \frac{g_t}{Ak_t^a} = \frac{d(1 - ab)}{(1 + d)} < 1 \tag{8}$$

which is a tax-smoothing result in a general equilibrium model.[11]

To summarise, the government's Markov strategy in (8), in combination with the private agent's optimal rules in (6) and (7), give a Markov-perfect general equilibrium. In this equilibrium, it is optimal to keep the tax rate constant over time.[12]

3. Empirical results

3.1 The econometric model

To test whether the general equilibrium model given by equations (6)-(8) is data consistent, we re-express each as a stochastic share of output[13]:

$$c_t / y_t = g_1(1-g_3) + m_{1t} \tag{9a}$$

$$k_{t+1} / y_t = g_2(1-g_3) + m_{2t}$$

(9b)

$$g_t / y_t = g_3 + m_{3t} \tag{9c}$$

where $g_1 = (1-ab)$, $g_2 = ab$, $g_3 = q$, and m_{it} for i=1,2,3 are stochastic shocks.[14] The single cross-equation overidentifying restriction implied by the tax-smoothing model in general equilibrium is $g_1 = 1-g_2$.

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[9] Because of (5), only one of the two policy instruments, q_t and g_t, can be set independently.

[10] By adding (6) and (7), we get $c_t+g_t+k_{t+1}=Ak_t^a=y_t$, i.e. the economy's resource constraint.

[11] That is, we derived a closed-form solution for the optimal tax rate in a general equilibrium model of growth and endogenous fiscal policy (for details, see Malley and Philippopoulos [1999]). Barro and Sala-i-Martin [1992], Benhabib and Velasco [1996] and Devereux and Wen [1998] have also derived closed-form solutions in similar setups. However, Benhabib and Velasco [1996] use a small open economy model in which the return to capital is determined by the exogenous world return, while Barro and Sala-i-Martin [1992] and Devereux and Wen [1998] use the *AK* model in which the return to capital, *A*, is a parameter. In contrast, in our model all returns are endogenously determined; thus, our setup is genuinely a general equilibrium one.

[12] Recall that, for simplicity, we consider only public consumption services. As stated above, our results do not change if we also include public production services. Assume that a portion $0<b<1$ of total tax revenues goes to public production services and the rest $0<1-b<1$ goes to public consumption services. Then, the optimal tax rate will again be constant over time, which in turn implies that the optimal public consumption services to output ratio, and the optimal public production services to output ratio, are also constant over time. That is, our main theoretical results do not change (this also generalizes Devereux and Wen [1998]). The same applies to our empirical results (see below). All these results are available from the authors on request.

[13] To obtain the shares we simply divided both sides of (6) and (7) by *y* and also use (8).

[14] To introduce a multiplicative stochastic shock (for instance, in the production function) in the theoretical model above is straightforward and does not change any of our results if we assume that agents make their decisions after the current shock is realized (see, e.g. Sargent [1987, pp. 51-55] and Stokey and Lucas [1989, p. 275]). However, when the shock enters additively (for instance, when the budget constraint in (2) is subject to an additive stochastic shock) the results change because the model is not linear-quadratic and hence certainty equivalence does not hold. For a similar problem in a linear-quadratic setup, see Lockwood *et al*. [1996, p. 904]). Nevertheless, we can show that, even when the shock enters additively, our main results do not change

3.2 Estimation and testing

We estimate and test the general equilibrium model (9a)-(9c), using annual data from 1960-1996 for all OECD economies.[15] Imposing the restrictions that are required for identification, we estimate (9a)-(9c) and check whether the remaining or overidentifying restriction is data consistent. We use the Full Information Maximum Likelihood (FIML) estimator. Relative to single equation estimators, the advantages of FIML in this context are that (i) it is generally more efficient and (ii) cross-equation restrictions can be implemented and tested.

Columns 3-5 of Table 1 below provide information pertaining to both the value and significance of the estimated model parameters. Column 6 reports the Wald test of whether the single cross-equation restriction is valid. The results in Table 1 reveal that some implications of the tax-smoothing model are supported by the data for all countries, e.g. both $q = \dfrac{d(1-ab)}{(1+d)}$ and $g_1 = (1-ab)$ are between zero and unity. However, the single cross-equation restriction imposed by the model is uniformly rejected for all 26 OECD economies, including separate aggregations for OECD and the 15 countries of the European Union.[16]

Therefore, although the tax-smoothing result has been one of the most popular models of endogenous fiscal policy, perhaps due to its clarity and algebraic convenience, our findings suggest that its empirical relevance is limited. Perhaps this finding is not surprising given the

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if we take an approximation around the deterministic version of the model. However, since this would unnecessarily complicate the theoretical model, we follow the usual practice and introduce shocks in the econometric model (9a)-(9c) in an *ad hoc* fashion.

[15] Data on private final consumption, C, public general consumption, G, and gross fixed capital formation, I, are from individual country Annual National Accounts. We access these accounts from the OECD Statistical Compendium 98(1). The end-of-period capital stock, K is calculated for each country using a perpetual inventory and a constant 7% rate of depreciation. Note that the results reported in Table 1 do not change when alternative deprecation rates ranging from 5 to 10% are employed.

[16] Application of recursive FIML estimation by using a variable start date with a fixed end date; a variable end date with a fixed start date; and a moving fixed window of 20 observations does not alter our main findings. To preserve space, these results are not presented here but can be made available on request.

very restrictive set of assumptions required to obtain the tax-smoothing result in a general equilibrium set-up. For instance, the model assumes fully rational and long-sighted behaviour on the part of private agents and policymakers, as well as full capital depreciation.

Table 1: Parameter Estimates & Wald Tests of Overidentifying Restrictions

Countries	*Estimation Period*	*Parameter Estimates & t-ratio* γ_1	γ_2	γ_3	Wald Test $\gamma_1 = (1-\gamma_2)$
OECD	1961-95	0.74 (396.6)	2.17 (23.4)	0.18 (90.7)	414.2
European Union	1961-95	0.73 (198.9)	2.33 (24.1)	0.20 (203.6)	424.9
West Germany	1961-95	0.70 (65.1)	2.58 (9.8)	0.20 (140.8)	68.9
France	1961-95	0.73 (83.3)	2.21 (12.6)	0.18 (50.3)	115.8
Italy	1961-95	0.71 (45.6)	2.42 (9.1)	0.19 (55.4)	57.3
Netherlands	1961-95	0.72 (112.9)	2.41 (21.3)	0.16 (80.7)	321.8
Belgium	1961-95	0.74 (394.6)	2.11 (18.6)	0.18 (77.3)	262.5
Luxembourg	1961-95	0.74 (400.7)	2.06 (20.5)	0.18 (84.5)	316.4
United Kingdom	1961-95	0.73 (219.9)	2.27 (21.2)	0.19 (209.2)	332.6
Ireland	1961-95	0.73 (232.5)	2.23 (21.3)	0.19 (201.5)	335.7
Denmark	1961-95	0.73 (208.9)	2.26 (21.5)	0.20 (198.0)	340.3
Spain	1961-95	0.73 (212.5)	2.25 (21.5)	0.19 (190.0)	339.4
Greece	1961-95	0.70 (62.2)	2.52 (7.92)	0.20 (145.2)	45.6
Portugal	1961-95	0.71 (28.5)	2.36 (4.49)	0.19 (27.0)	14.2
United States	1961-95	0.79 (217.2)	1.68 (18.9)	0.08 (35.1)	275.3
Mexico	1961-95	0.76 (138.9)	1.89 (20.0)	0.22 (38.8)	322.7
Republic of Korea	1971-95	0.67 (50.6)	1.93 (13.6)	0.13 (25.7)	152.6
Japan	1961-95	0.68 (125.6)	2.40 (17.3)	0.11 (42.5)	237.2
Australia	1961-95	0.72 (119.4)	2.36 (6.6)	0.16 (22.3)	32.3
Norway	1961-95	0.66 (72.9)	2.94 (8.2)	0.19 (16.4)	50.1
New Zealand	1961-95	0.77 (127.8)	1.99 (2.7)	0.16 (10.4)	5.81
Sweden	1961-95	0.74 (283.8)	2.35 (19.7)	0.26 (57.6)	300.2
Finland	1961-95	0.66 (108.3)	2.94 (20.2)	0.20 (46.6)	302.8
Iceland	1961-95	0.73 (68.6)	2.21 (10.1)	0.16 (12.1)	72.9
Switzerland	1961-95	0.71 (183.1)	2.57 (16.5)	0.13 (62.2)	212.3
Austria	1961-95	0.71 (301.2)	2.39 (21.7)	0.21 (102.2)	367.3
Turkey	1961-95	0.80 (87.5)	1.47 (16.9)	0.09 (35.4)	242.7

Note: the critical value of the Wald test (which is distributed χ^2) for one degree of freedom at the 5% significance level is 3.84.

4. Conclusions and future work

In this paper we have constructed a general equilibrium model of optimal growth and endogenous fiscal policy in which policymakers find it optimal to keep the tax rate constant over time. However, data from 26 OECD economies uniformly reject the empirical viability of this model. In contrast to the findings from the partial equilibrium studies cited above, our results suggest that the policy recipe to keep the tax rate flat over time does not hold when private and public agents are allowed to react to each other. This is consistent with the findings of Chari et al. [1994] for the U.S.. Since the tax-smoothing result relies on some rather unrealistic assumptions about the functioning of the economy, in related work we search for alternative general equilibrium models of optimal growth and endogenous fiscal policy, which may be more consistent with the data (see Malley and Philippopoulos [1999]). In particular, our preliminary findings suggest that setups which allow for deviations from the assumptions of full rationality are much more data friendly.

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