Environmental negotiations as dynamic games: Why so selfish?

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SUMMARY

We study a trade-off between economic and environmental indicators using a two-stage optimal control setting where the player can switch to a cleaner technology, that is environmentally "efficient", but economically less productive. We provide an analytical characterization of the solution paths for the case where the considered utility functions are increasing and strictly concave with respect to consumption and decreasing linearly with respect to the pollution stock. In this context, an isolated player will either immediately start using the environmentally efficient technology, or *for ever* continue applying the old and "dirty" technology. In a two-player (say, two neighbor countries) dynamic game where the pollution results from a sum of two consumptions, we prove existence of a Nash (open-loop) equilibrium, in which each player chooses the technology selfishly *i.e.*, without considering the choice made by the other player. A Stackelberg game solution displays the same properties. Under cooperation, the country reluctant to adopt the technology as an equilibrium solution, chooses to switch to the cleaner technology provided it benefits from some "transfer" from the environmentally efficient partner.

KEY WORDS: Pollution, technology adoption, optimal control, dynamic games

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1. Introduction

The trade-off between economic performance and environmental efficiency is becoming a key aspect of economic policy debates at all levels, as it transpires from the discussion around the Kyoto protocol. Indeed, for many authors, such trade-offs exist, and dealing with them is a matter of urgency. For example, [1] have found a decreasing relationship between economic performance, as measured by the return to capital employed, and a composite environmental indicator computed from emissions of SO_2 and NO_x, and chemical demand for oxygen in German, Italian, Dutch and British industries over the period 1995-1997. In a context of some specific industries, at an earlier date [2] have argued that the share of environmental costs in *total* manufacturing costs might well be a considerable burden, and is by no way offset by any kind of advantage.

Such evidence is at odds with the so-called Porter hypothesis (see [3], [4]), which advocates a kind of win-win situation induced by more stringent environmental norms (see a game-theoretic foundation for Porter hypothesis in [5]. However, for most environmental economists, it seems clear that enforcing stricter environmental norms should negatively affect economic performance, at least in the short run. Whether such a regulation could induce further innovations and generate at some point in time highly efficient technologies in all respects (see for example [6]) is not discussed in this paper. We restrict our attention to finite and relatively short time horizons. In such a context, a trade-off between economic performance and environmental efficiency is hardly questionable.

Beside the empirical arguments, there might be several mechanisms leading to such a trade-off. Of course, one should first mention the direct output losses due, for example, to more stringent emission norms (as in the Kyoto protocol, see [7], for a discussion). In the short run, an additional economic inefficiency is likely to arise if the compliance to more stringent environmental norms implies costly development and adoption of new and less polluting technologies. In this paper, we abstract from R&D efforts and use output losses to proxy cost increases. In a benchmark case we consider a country that can continue using an old technology or switch to a cleaner but less productive technology (because of the involved adoption costs). Adoption costs are usually associated with a transitory or permanent fall in productivity (see a survey in [8]), and we shall also use this argument in our context.

Here we sketch a base for our model. Adopting a new technology can be immediate or delayed. Individuals populating the economy dislike pollution and enjoy large output. (Hence, a trade-off arises formally because one unit of output could be sacrificed for *so-many* units of abated pollution.) Furthermore, we do not include any compulsory environmental norms in the model, and allow choices to be made on the basis of a simple trade-off model. In the benchmark case, the problem will be formulated as a two-stage optimal control problem as in [9], who extended previous contributions of [10] and [11]. In this paper, we shall go a step further and embed the benchmark two-stage optimal control problem in a two-country game-theoretic context.

We shall show that whether the less polluting technology is adopted or not, relies heavily on the strategic ingredients of our model like the data on marginal productivity and marginal propensity to pollution of the new technology. The 2006 Canadian decision to withdraw from the Kyoto protocol and to join the opposing club is a clear signal of the ongoing tensions and strategic steps taken all over the world, in this specific field. These tensions reflect the pressure of many national and international lobby groups. In the case of Canada, the manufacturing sector as represented by the CME (Canadian Manufacturers and Exporters), has made repeatedly clear that the commitments and the measures adopted to meet Kyoto requirements must "...be part of a meaningful international strategy for limiting atmospheric greenhouse gases ... and lead to genuine reductions in greenhouse gas emissions that are measurable, verifiable, practical, and economically feasible...".* Clearly, the US decision to

*See the web site at http://www.cme-mec.ca/kyoto/.

act unilaterally and not ratify the Kyoto protocol is strong evidence that this country does not perceive *optimal* joining the Kyoto club. Undoubtedly, this decision can only encourage self-interested behavior of other countries.

Here is a brief outline of how this paper is organized. The next section solves the benchmark problem of an isolated country facing a short-run trade-off between an economic and an environmental efficiency. Section 3 considers a two-country open-loop Nash game and also comments on a leader-follower version of the game. Section 4 studies the outcomes of cooperative games. Section 5 provides concluding remarks.

2. The benchmark optimal control problem

2.1. The model

In this section, we consider the case of an isolated economy, which therefore takes its decisions in exclusive accordance with its own preferences and constraints. For this case, we shall develop the computations in detail.

Consider an economy whose benevolent central planner wants to maximize the following intertemporal utility function:

$$\int_0^T u\left(C(t), P(t)\right)\ e^{-\rho t} dt, \tag{1}$$

where C is aggregate consumption, P is the aggregate stock of pollution, and $u(.)$ is a concave utility function with $u_C^{'} > 0$ et $u_P^{'} < 0$. The time horizon is T, assumed finite in this framework and ρ is the discount rate.

It should be noted here that adding a scrap value (as a function of the stock of pollution at the terminal date T for example) will not change the main results of the paper. *

On the production side, we have an elementary one-sector structure: the production function

*All formulae derivations are available upon request from the authors.

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is assumed to be of the AK type, and output is either used for consumption or as an input, X:

$$Y = C + X = F(X) = A_i\ X \Rightarrow C \equiv X(A_i - 1) \tag{2}$$

where $A_i > 1$ is the marginal productivity of input in technology i. We will restrict our attention in this paper to $i = 1, 2$ where $i = 1$ refers to the "current" technology and $i = 2$ to the "new" technology.

The stock of pollution is assumed to evolve proportionally to the production level:

$$\dot{P} = \alpha_i A_i X, \tag{3}$$

with $P(0) \geq 0$ given. As $A_i X$ is output, α_i measures the marginal contribution to pollution of an additional production unit. Clean technologies would therefore be associated with low values of α_i and highly productive technologies would have large A_i.

As said after (1), we assume that $P(T)$ is free* in this benchmark case. Given a pollution objective at the terminal date T, we could have assumed that $P(T)$ is given. This would not affect our results.*

Hereafter, we shall represent any technological menu by a pair of positive numbers (A_i, α_i). We assume that at $t = 0$ the economy is equipped with technology (A_1, α_1) but another menu (A_2, α_2) is also available with $\alpha_1 > \alpha_2 > 0$ but $1 < A_2 < A_1$.

Here we mean several things. Cleaner technologies, presumably the "new" technologies, involve a number of adoption costs (see [8], for a survey). For example productivity may not be high at the early stages of the implementation of this new technique (as empirically documented by [12]) until enough specific human capital is accumulated (as in the typical learning-by-doing model, see [13]). Of course, such productivity losses may only be transitory,

*Notice that, given the law of motion of the stock of pollution, and $P(0) \geq 0$, we have necessarily $P(t) \geq 0$, $\forall t \geq 0$. Hence, $P(T) > 0$.

*For transparency, we have decided to not include in this article many bulky algebraic derivations but, of course, they are available upon request.

as explicitly accounted for by [9]. Because of that we consider finite horizon optimization problems in this paper. This will allow us to argue that the above mentioned productivity losses last for the whole horizon considered and can be modeled through one parameter A_2. Hence our claim of solving a rather realistic problem of a government elected for the finite period T.

The stylized government problem, which we solve in this paper, consists of deciding whether the economy remains using the technology menu "1" or adopts the new technological regime "2", with less pollution ($a_2 < a_1$) at the expense of less productivity ($A_2 < A_1$), during its ruling period. If the latter alternative is true, the government needs to announce the optimal switching time, say t_1, with $0 \leq t_1 \leq T$.

Mathematically, the optimal control problem that we endeavor to solve is

$$\max_{X,t_1} \left\{ U(X,P,t_1) \equiv \int_0^T u(X(t),P(t))e^{-\rho t} dt \right\}, \tag{4}$$

subject to (8) and (2), P_0 given and $P(T)$ free.

Notice that we can rewrite our objective function as:

$$U(X,P,t_1) = \int_0^{t_1} u(X,P)e^{-\rho t} dt + \int_{t_1}^T u(X,P)e^{-\rho t} dt \tag{5}$$

where menu "1" is used in $[0, t_1)$; at t_1 menu "2" is adopted and applied until the end of horizon T.

In order to characterize our results analytically, we shall restrict our study to the following class of utility functions:

$$u(C,P) = \ln(C) - \beta\, P,$$

where β measures the marginal disutility due to pollution, which is assumed in our analytical case independent of the level of the pollution stock.

2.2. Solving the optimal control problem

We solve the two-stage optimal control problem of technology adoption (4) backward in time, as in dynamic programming (compare [10] and [11]), starting with the second possible regime.

The calculations are elementary but rather bulky. We shall give the algebraic details of the solution derivation in the benchmark case. The other cases are dealt with in the similar fashion.

2.2.1. Control on $[t_1, T]$ We assume that the economy will switch to the new technology at t_1. If so, the optimal control problem the switch is:

$$\max_X U_2(X, P, t_1) = \int_{t_1}^{T} (\ln(X(t)(A_2 - 1)) - \beta P)\, e^{-\rho t} dt \tag{6}$$

subject to $\dot{P} = \alpha_2 A_2 X$, with $P(t_1) = P_1 \geq 0$ given and free $P(T)$. We will use the Pontriagin's *minimum* principle to establish an optimal solution.

The corresponding Hamiltonian is defined as:

$$H_2(P, X, t, \lambda_2) = -e^{-\rho t} (\ln(X\ (A_2 - 1)) - \beta P) + \lambda_2\ (\alpha_2 A_2\ X),$$

and the first-order conditions are:

$$\frac{-e^{-\rho t}}{X} \quad + \quad \lambda_2 \alpha_2 A_2 = 0 \tag{7}$$

$$\dot{P} \quad = \quad \alpha_2\ A_2\ X \tag{8}$$

$$\dot{\lambda}_2 \quad = \quad -e^{-\rho t}\beta\,, \qquad \lambda_2(T)\, P(T) = 0\,. \tag{9}$$

It follows from (9) that

$$\lambda_2(t) = \frac{e^{-\rho t}\beta}{\rho} + c, \tag{10}$$

where c is a constant that will be determined using the transversality condition. Since $P_1 \geq 0$, thus $P(T) > 0$ by the law of motion of the stock of pollution and the transversality condition becomes $\lambda_2(T) = 0$. This yields

$$c = -\frac{e^{-\rho T}\beta}{\rho},$$

and, finally,

$$\lambda_2(t) = \frac{\beta}{\rho} \left(e^{-\rho t} - e^{-\rho T}\right). \tag{11}$$

We can now calculate the remaining variables.

The first-order condition with respect to X (*i.e.*, (7)) implies that

$$\alpha_2 A_2 \, X(t) = \frac{e^{-\rho t}}{\lambda_2(t)},$$

which yields, using the law of motion (3)

$$P(t) = -\frac{1}{\beta} \ln \left(e^{-\rho t} - e^{-\rho T} \right) + c_2 \tag{12}$$

where c_2 is a constant given by the initial condition, $P(t_1)$, that is:

$$P_1 \equiv P(t_1) = -\frac{1}{\beta} \, \ln \left(e^{-\rho t_1} - e^{-\rho T} \right) + c_2$$

And so

$$c_2 = P_1 + \frac{1}{\beta} \, \ln \left(e^{-\rho t_1} - e^{-\rho T} \right) \, .$$

The pollution accumulation path is then

$$P(t) \quad = \quad P_1 + \frac{-\ln \left(e^{-\rho \, t} - e^{-\rho \, T} \right) + \ln \left(e^{-\rho \, t_1} - e^{-\rho \, T} \right)}{\beta} \tag{13}$$

and the corresponding optimal input path

$$X(t) = \frac{1}{\alpha_2 A_2} \cdot \frac{\rho}{\beta} \cdot \frac{1}{1 - e^{-\rho(T-t)} \, , \quad t \in [t_1, T]} \tag{14}$$

For the discussion on existence of t_1 such that $0 < t_1 < T$, we will need the expression of the optimal Hamiltonian for the "second" regime at any date between t_1 and T. After some algebra, the optimal Hamiltonian $H_2^*(P_1, t_1)$ equals

$$H_2^*(P_1, t_1) = e^{-\rho t_1} \left(1 - Q_2 + \rho t_1 + \beta P_1 + \ln(e^{-\rho t_1} - e^{-\rho T}) \right)$$

where

$$Q_2 \equiv \ln(\rho) - \ln(\alpha_2 A_2 \beta) + \ln(A_2 - 1) \, . \tag{15}$$

We notice that formulae (14) and (13) derived for $t \in [t_1, \, T]$ are valid for any $t_1 \geq 0$ (*i.e.*, including $t_1 = 0$). This enables us to formulate a corollary regarding the optimal behavior of the central planner for any uninterrupted control period.

Corollary 1. *A central planner that is maximizing a discounted stream of differences between utility from consumption and disutility from pollution, expressed as (6), chooses an increasing input path (14), which is inverse proportional to the pollution accumulation speed. The resulting pollution accumulation path is hence independent of the production and pollution technologies.*

We infer from the corollary that “no matter” the damage caused by pollution, measured by $\int_{t_1}^{T} e^{-\rho t}\beta P(t)dt$, the planner is able to choose an input path $X_i(t)$ (and hence consumption $(A_i - 1)X(t)$) that maximizes the “trade-off” utility function (6).

2.2.2. Control on $[0,\, t_1]$ Consider (4) and (5). We have already solved the problem on $[t_1,\, T]$. Given optimal control (14) and pollution stock at t_1 (*i.e.*, $P(t_1)$) we can calculate the optimal utility for this part of the horizon; will will denote it a $U_2^*(P_1, t_1)$. Hence, to compute optimal control on $[0,\, t_1]$, we have to solve the following problem

$$\max_{\{X,\ t_1\}} U(X,P,t) = \int_0^{t_1} \left(\ln(X(t)\,(A_1-1)) - \beta P(t)\right) e^{-\rho t} dt + U_2^*(P_1,t_1) \qquad (16)$$

subject to $\dot{P}\,(t) = \alpha_1 A_1 X(t)$, $0 \le t < t_1$, with P_0 given and $P_1 = P(t_1)$ free. Hence the old technology problem (menu “1”) is with free end point and free terminal time.

The corresponding Hamiltonian* is

$$H_1(P,X,t,\lambda_1) = -e^{-\rho t}\left(\ln(X\,(A_1-1)) - \beta P\right) + \lambda_1\,(\alpha_1\ A_1 X),$$

and the corresponding first-order conditions:

$$\frac{-e^{-\rho t}}{X} \;+\; \lambda_1\alpha_1 A_1 = 0 \qquad (17)$$

$$\dot{P} \;=\; \alpha_1\ A_1\ X \qquad (18)$$

$$\dot{\lambda}_1 \;=\; -e^{-\rho t}\beta\,. \qquad (19)$$

Following [10] we use

$$\lambda_1(t_1) = \lambda_2(t_1) \qquad (20)$$

*Actually, for problems with free terminal time, an *extended* Hamiltonian is needed *i.e.*, one which includes a costate responsible for the time variable. However, [10] shows that this in not necessary in this case.

(and also $H_1(t_1) = H_2(t_1)$, see (27) below) and compute the shadow price as

$$\lambda_1(t) = \frac{\beta}{\rho}\left(e^{-\rho t} - e^{-\rho T}\right). \tag{21}$$

Notice that (21) does not depend on t_1. This is a result of using (20) to calibrate an indefinite integral of (19).

We can formulate the following corollary by comparing (21) to (11).

Corollary 2. *The pollution shadow price $\lambda(t)$ is independent of the technological menu i.e., it is the same for each technological choice.*

This means that a central planner that is maximizing a discounted stream of differences between utility from consumption and disutility from pollution, expressed as (6), perceives disutility due to the accumulated pollution $P(t)$ as a (decreasing) function of time only.

Now, just like in the new technology regime problem (menu “2”, treated in Section 2.2.1), we can find the optimal path for the stock of pollution, $P(t)$, and the production input $X(t)$, $t \in [0, t_1]$. After some easy algebra, we get:

$$P(t) = -\frac{1}{\beta}\ln\left(e^{-\rho t} - e^{-\rho T}\right) + c_1, \tag{22}$$

where c_1 is a constant, which results from the initial condition $P_0 = P(0)$:

$$c_1 = P_0 + \frac{1}{\beta}\ln\left(1 - e^{-\rho T}\right). \tag{23}$$

Hence,

$$P(t) = P_0 + \frac{1}{\beta}\ln\left(\frac{1 - e^{-\rho\, T}}{e^{-\rho\, t} - e^{-\rho\, T}}\right), \quad t \in [0, t_1]\,. \tag{24}$$

and the optimal input for $X(t)$ is given by:

$$X(t) = \frac{1}{\alpha_1 A_1}\cdot\frac{\rho}{\beta}\cdot\frac{1}{1 - e^{-\rho(T-t)}}, \quad t \in [0, t_1]. \tag{25}$$

This is sufficient for us to compute the optimal value of intertemporal utility as a function of (P_1, t_1) and also of the Hamiltonian corresponding to the old technology regime. In particular, the value of the Hamiltonian at the P_1, t_1

$$H_1(P_1, t_1) = -e^{-\rho t_1}\left(\ln\left(X(t_1)\left(A_1 - 1\right)\right) - \beta P(t_1)\right) + \lambda_1(t_1)(\alpha_1 A_1 X(t_1))$$

can be expresses, after some algebra, as

$$H_1^*(P_1,t_1) = e^{-\rho t_1}\left(1 - Q_1 + \rho t_1 + \beta c_1\right),$$

where

$$Q_1 = \ln(\rho) - \ln(\alpha_1 A_1 \beta) + \ln(A_1 - 1)\,. \tag{26}$$

and c_1 is as in (23).

We are now ready to answer the crucial question about an optimal interior switching date t_1.

2.2.3. Existence of optimal t_1 One can determine the optimal switching time from the optimality condition, see [10]:

$$H_2^*(P_1,t_1) - H_1^*(P_1,t_1) = 0 \tag{27}$$

where both Hamiltonian values were computed previously as:

$$H_1^*(P_1,t_1) = e^{-\rho t_1}\left(1 - Q_1 + \rho t_1 - \beta c_1\right)$$
$$H_2^*(P_1,t_1) = e^{-\rho t_1}\left(1 - Q_2 + \rho t_1 - \beta c_2\right)\,.$$

This yields

$$e^{-\rho t_1}\left(Q_1 - Q_2\right) = 0\,. \tag{28}$$

However, this means that *there is no interior switching time* unless $Q_1 = Q_2$, see (15) and (26). In that case the planner is indifferent between the two technological regimes; moreover, t_1 can take any value on $[0,T]$.

Corner solutions (*i.e.*, starting the new technology either at time 0 or never) were theoretically pointed out in in [10]] and in [11]. According to (28) the economy should postpone the new technology adoption indefinitely (*i.e.*, $t_1 \to \infty$).

The following corollary is a consequence of (28).

Corollary 3. *The sufficient and necessary condition for never-adopting the new technology is* $Q_1 > Q_2$ *that means*

$$\text{(NA)} \qquad \frac{\alpha_2 A_2}{A_2 - 1} > \frac{\alpha_1 A_1}{A_1 - 1}. \tag{29}$$

Symmetrically, the sufficient and necessary condition for an immediate adoption of the new technology is $Q_1 < Q_2$ *that means*

$$\text{(IA)} \qquad \frac{\alpha_2 A_2}{A_2 - 1} < \frac{\alpha_1 A_1}{A_1 - 1}. \tag{30}$$

The above conditions have a simple economic interpretation. A marginal increase in input X increases output by A_i and consumption by $A_i - 1$ and contributes to the stock of pollution by $\alpha_i A_i$. Condition (NA) says that the economy should never adopt the new technology, which is less polluting ($a_2 < a_1$) but also less productive ($A_2 < A_1$) than the old one, if the ratio of marginal pollution to marginal consumption for the new technology *dominates* the ratio calculated for the old technology.

If (IA) holds, then the economy should immediately adopt the less polluting technology that guarantees a lower ratio of marginal pollution to marginal consumption.

In simple terms, the implemented technology assures a high consumption *i.e.*, $A_i - 1$ will be large, or a low pollution *i.e.*, $\alpha_i A_i$ will be small (or both).

3. A two-country problem

3.1. The model

Assume that there are two countries a and b that suffer from the same pollution stock. This is a frequent case of countries with a common border that are using similar production technologies. Similarly to [could you cite?.. you have written that this "is a usual assumption in related papers" ..] we shall assume that these countries do not trade in goods. This will help analytical tractability of the problem. Nevertheless, we believe that countries' trade may be limited if their economies' products are substitutes rather than complements. Again, this is non-unfrequent among neighboring regions.

As before, there is a central planner in each economy whose intertemporal utility function is

$$\int_0^T u^k(C^k(t), P(t))e^{-\rho t}dt,$$

with $k \in \{a, b\}$. The production function, in the consumption goods sector, is AK; output is used either for consumption or as input X, as follows:

$$Y^k = C^k + X^k = F^k(X^k) = A_i^k X^k \quad \Rightarrow \quad C^k \equiv X^k(A_i^k - 1)$$

where $A_i^k > 1$ is marginal productivity of input in country k.

The main difference with respect to the benchmark case is in the evolution of the pollution stock, which now follows

$$\dot{P} = \alpha_i^a A_i^a X^a + \alpha_i^b A_i^b X^b \,. \tag{31}$$

That is, both countries contribute to the common stock of pollution depending on the level of their production and technology in use.

The technological menus accessible to each countries are indexed "i, k" hence need not be the same. This might be due to different technological innovation and/or absorption capacities. However, menu "2" may be identical for each country.

We shall keep the planners' utility functions identical and equal to the utility function dealt with in the benchmark case *i.e.*,

$$u^k(C, P) = \ln(C) - \beta^k \; P,$$

where β^k can differ between the countries. We will now study how the benchmark solution identified in Section 2 is altered by the two-country context.

We believe that although knowledge about disutility from pollution $X(t)$ is well developed in each country, the actual *measurements* of $P(t)$ are difficult and largely unreliable. This enables us to restrict the analysis to open-loop Nash equilibria. As it is well known, the open-loop Nash equilibrium in an N-players differential game is found by solving N optimal control problems taking as given the strategies of other players.

3.2. A two-stage open-loop Nash equilibrium

3.2.1. The optimization problem of player a Player a seeks her best strategy, given the strategy of the other player. For simplicity we suppose that the other player keeps the technology unchanged for the entire optimization horizon $[0, T]$.* Because most of the (rather involved) computations are very similar to those of the benchmark case, we will skip most of the intermediate steps.

The maximization problem of player a is:

$$\begin{aligned} \max_{X,t_1} U^a(X^a, P, t_1) &= \int_0^{t_1} \left(\ln\left(X^a(A_1^a - 1)\right) - \beta P\right) e^{-\rho t} dt \\ &+ \int_{t_1}^{T} \left(\ln\left(X^a(A_2^a - 1)\right) - \beta P\right) e^{-\rho t} dt \end{aligned} \tag{32}$$

subject to

$$\dot{P} = \alpha_2^a A_2^a X^a(t) + \alpha^b \ A^b \ X^b(t), \quad \text{if} \quad t \geq t_1; \tag{33}$$

$$\dot{P} = \alpha_1^a A_1^a X^a(t) + \alpha^b \ A^b \ X^b(t), \quad \text{if} \quad t < t_1\,, \tag{34}$$

with P_0 given, $P(T)$ free, and $X^b(t)$ given.

This problem is similar to the benchmark problem with the unique difference that the state equation now involves an *additive* forcing term $\alpha^b \ A^b \ X^b(t)$. However, the contribution of this term to optimal decisions of player a is null because the derivatives of the Hamiltonian of this player do not depend on X^b. Hence, the outcomes of the benchmark model are also valid in this case. This becomes clear by the following derivations.

We proceed to solve the problem of player a beginning form period $[t_1, T]$, as in Section 2.

*his assumption could easily be relaxed and the main results generated in that case would not be altered; see [14].

3.2.2. Control on $[t_1, T]$ Suppose that country a decides to adopt the new technology at date t_1 given the steady play by country b as described above. So, the problem of country a is to maximize

$$H_2^a = -e^{-\rho t}\left(\ln(X^a(A_2^a-1)) - \beta^a P\right) + \lambda_2^a(\alpha_2^a A_2^a X^a(t) + \alpha^b A^b X^b(t))$$

where α^b, A^b and $X^b(t)$ are given. The first-order conditions are:

$$\frac{e^{-\rho t}}{X^a} = \lambda_2^a \alpha_2^a A_2^a \tag{35}$$

$$\dot{P} = \alpha_2 A_2^a X^a + \alpha^b A^b X^b \tag{36}$$

$$\dot{\lambda}_2^a = -e^{-\rho t}\beta^a \tag{37}$$

$$\lambda_2^a(T) = 0\,. \tag{38}$$

Using the transversality condition, one can obtain, as in the benchmark case, the solution path for the co-state variable:

$$\lambda_2^a(t) = \frac{\beta^a}{\rho}\left(e^{-\rho t} - e^{-\rho T}\right)\,. \tag{39}$$

Furthermore, we compute the time profiles of $P(t)$ and $X^a(t)$ as follows:

$$P(t) = -\frac{1}{\beta^a}\ln\left(e^{-\rho t} - e^{-\rho T}\right) + \alpha^b A^b Z(t) + \tilde{p},$$

where $Z(t)$ the integral of the other country's control $X^b(t)$, defined as $\int_t^T X^b(t)dt$ and $\tilde{p}$ is a constant determined by $P(t_1)$ *i.e.*, the amount of pollution accumulated at time t_1:

$$\tilde{p} = P_1 + \frac{1}{\beta^a}\ln\left(e^{-\rho t_1} - e^{-\rho T}\right) - \alpha^b A^b Z(t_1)\,.$$

The optimal path of $X^a(t)$ is given by

$$X^a(t) = \frac{1}{\alpha_2^a A_2^a}\cdot\frac{\rho}{\beta^a}\cdot\frac{1}{1-e^{-\rho(T-t)}}\,. \tag{40}$$

Finally, we can write the optimal Hamiltonian $H_2^a(.)$; in particular at the assumed switching time t_1, it equals to

$$\begin{aligned} H_2^a(P_1,t_1) &= e^{-\rho t_1}\left(1 - Q_2^a + \rho t_1 + \beta^a\tilde{p} + \beta^a\alpha^b A^b\left(\frac{X^b(t_1)}{\rho} + Z(t_1)\right)\right) \\ &- \frac{\beta^a}{\rho}\alpha^b A^b X^b(t_1)e^{-\rho T}, \end{aligned} \tag{41}$$

where $Q_2^a = \ln(\rho) - \ln(\alpha_2^a A_2^a \beta^a) + \ln(A_2^a - 1)$.

3.2.3. Control on $[0,\, t_1]$ The second step of the solution procedure works as in Section 2.2.2. The Hamiltonian associated with the old technology used from 0 to t_1 is:

$$H_1^a = -e^{-\rho t}\left(\ln(X^a(A_1^a - 1)) - \beta^a P\right) + \lambda_2^a(\alpha_1^a A_1^a X^a(t) + \alpha^b A^b X^b(t)),$$

with α^b, A^b and $X^b(t)$ given. This optimal control problem has the same first-order conditions as the problem corresponding to the new technology *i.e.*, (35) to (37) with menu "1" replacing menu "2". Obviously, the transversality condition is different than (38). Instead, one has to use the initial stock of pollution at date 0, and some continuity properties. Allowing for those, one gets

$$\lambda_1^a = \frac{e^{-\rho t}\beta^a}{\rho} + c_1^a, \tag{42}$$

where c_1^a is

$$c_1^a = -\frac{e^{-\rho T}\beta^a}{\rho} \tag{43}$$

and was computed from $\lambda_1^a(t_1^-) = \lambda_2^a(t_1^+)$. Using (42) and (43) jointly yields

$$\lambda_1^a(t) = \frac{\beta^a}{\rho}\left(e^{-\rho t} - e^{-\rho T}\right). \tag{44}$$

The solution paths for the stock of pollution, production input and the resulting optimal Hamiltonian value at the switching date t_1 are then successively computed as:

$$P(t) = -\frac{1}{\beta^a}\ln\left(e^{-\rho t} - e^{-\rho T}\right) + \alpha^b A^b Z(t) + \hat{p}, \tag{45}$$

$$X^a(t) = \frac{1}{\alpha_1^a A_1^a}\cdot\frac{\rho}{\beta^a}\cdot\frac{1}{1 - e^{-\rho(T-t)}} \tag{46}$$

$$\begin{aligned} H_1^{a,*}(P_1, t_1) &= e^{-\rho t_1}\left(1 - Q_1^a + \rho t_1 + \beta^a\hat{p} + \beta^a\alpha^b A^b\left(\frac{X^b(t_1)}{\rho} + Z(t_1)\right)\right) \\ &- \frac{\beta^a}{\rho}\alpha^b A^b X^b(t_1)e^{-\rho T}, \end{aligned} \tag{47}$$

where $Z(t)$ is the same integral of $X^b(t)$ but calculated for the period when the first technology is used as $\int_0^t X^b(t)dt$ (so, $Z(0) = 0$). We denote $Q_1^a = \ln(\rho) - \ln(\alpha_A^a A_A^a \beta^a) + \ln(A_1^a - 1)$; a constant determined by an initial condition on $P(0)$ is $\hat{p}$:

$$\hat{p} = P_0 + \frac{1}{\beta^a} \ln\left(1 - e^{-\rho T}\right) .$$

Using the definitions of $\tilde{p}$, $\hat{p}$, and P_1 jointly leads to $\hat{p} = \tilde{p}$ and so

$$P_1(t) = P_0 + \frac{1}{\beta^a} \ln\left(\frac{1 - e^{-\rho T}}{e^{-\rho t_1} - e^{-\rho T}}\right) + \alpha^b A^b Z(t_1).$$

3.2.4. Existence of optimal t_1 It is now possible to derive the optimality condition with respect to the switching time t_1, an interior optimum would arise if and only if

$$H_2 - H_1 = 0$$

This means

$$\begin{aligned} & e^{-\rho t_1}\left(1 - Q_2^a + \rho t_1 + \beta^a \tilde{p} + \beta^a \alpha^b A^b \left(\frac{X^b(t_1)}{\rho} + Z(t_1)\right)\right) - \frac{\beta^a}{\rho}\alpha^b A^b X^b(t_1) e^{-\rho T} \\ & - e^{-\rho t_1}\left(1 - Q_1^a + \rho t_1 + \beta^a \hat{p} + \beta^a \alpha^b A^b \left(\frac{X^b(t_1)}{\rho} + Z(t_1)\right)\right) - \frac{\beta^a}{\rho}\alpha^b A^b X^b(t_1) e^{-\rho T} = 0, \end{aligned}$$

and leads us to the same condition as in the one country case:

$$e^{-\rho t_1}\left(Q_1^a - Q_2^a\right) = 0.$$

Therefore, the unique Nash equilibrium here obtained generates a case in which each country plays "selfish" *i.e.*, independently of what the other country is doing.

3.3. Discussion

The above result may be found surprising. To explain it, we first formulate a corollary based of the comparison of (44) to (39).

Corollary 4. *The pollution shadow price $\lambda(t)$ is independent of the technological menu of either country.*

So, as in Corollary 2, we observe that solving the "trade-off" optimization problem (32)-(34) has led us to input strategies that "compensate" the damage done by pollution so that the shadow price of the latter depends on the running time only, and not on a technological regime. Hence, the trade-offs in the Nash equilibrium are similar to those faced by an isolated country.

As said in the above corollary, the marginal pollution cost is independent of whether the stock of pollution has increased by the action of other player. For any production profile of country b, the stock of pollution increases by $\alpha_i^a A_i^a$, if country a rises its production input by one unit. In equilibrium, each country's contribution to the stock of pollution rises by ρ/β_k multiplied by an increasing time-dependent weight and this is how each country "manages" the growing pollution stock.

We notice that Corollary 3 is also applicable to the two-country context and, in particular, the conditions (NA) and (IA) determine whether a country adopts a new technology immediately or never.

Nonetheless, one would think that the willingness to adopt a new technology should be different in the two-country case. This could be because the other player also contributes to the total stock of pollution, which should induce some strategic interaction to the game. We propose the following framework to analyze this issue now.

We have assumed in our calculations that player b follows a corner regime *i.e.*, either chooses the new technology immediately or never. That it is: $\alpha^b(t) = \alpha^b$ and $A^b(t) = A^b$, for all t. Now, we consider a game with $\alpha^b(t), A^b(t)$ possibly varying in time. By construction, if the coefficients vary, they are piecewise constant with one discontinuity point, say t', which would be the technology adoption date by country b. Denoting by $Z(t)$ an indefinite integral of function $\alpha^b(t)A^b(t)X^b(t)$ (instead of just $X^b(t)$, as before) we would obtain the optimality condition for an interior switching time of country a that would show the one-sided limits of $\alpha^b(t)$, $A^b(t)$, $X^b(t)$ and $Z(t)$ to the left and to the right of the switching point of country a, t_1. We could prove that all these terms would vanish because these functions are piecewise constant... unless t_1 is also the switching date of the country b. This problem was studied as a special case in [14] whose findings are summarized in Section 3.4.

However, in a non-cooperative game, studied in this section, the general result is that each country plays "selfishly" *i.e.*, independently of what the strategy of the other country is.

Of course, our result relies on the simple model that allowed us to obtain an analytical characterization of the solutions. However, we believe that our model and results capture recent behavior of some countries. Clean technologies are often rejected by countries which typically put forward the resulting fall in production (and thus in consumption) as the reason. In terms of our model, such countries are not willing to switch to cleaner technology because they are able to rise consumption under the old technology to "compensate" the disutility caused by increasing pollution. A parallel explanation is that the resulting relative fall in consumption would be valued much higher than the subsequent relative gain in pollution control. Typically, those countries are insensitive to the fact that other countries (even neighboring countries with common resources) have or have not chosen to adopt (partially or totally) cleaner technologies, which is predicted by our model. The reverse holds too.

In such a context, a question arises whether a cooperative (or "efficient") solution exists. Our model can deliver some simple conclusions also in this respect, see Section 4. However, we first comment on a leader-follower game.

3.4. A leader-follower game

In [14] the two-country game was modeled as a leader-follower problem. We refer to that paper for the solution derivations. Here, we summarize the findings.

Corollary 5. *In a two-country open-loop Stackelberg game of technology adoption, the pollution shadow prices are independent of the technological menu of either country.*

This conclusion leads to a set of sufficient conditions for technology adoption that are identical with (IA) and (NA). The leader will play "selfishly", whatever the behavior and characteristics of the follower and so will play the follower.

It appears that in technology adoption games modeled as trade-off problems, cooperation may be the only way to prevent agents' selfish decisions. We examine this issue in the next section.

4. An efficient solution

4.1. The model and solution

We assume that if the two countries adopt the new technology then they do it at the same time and suppose that the planner defines a symmetric Pareto solution as "efficient". The resulting cooperative game is

$$\begin{aligned} &\max_{X^a, X^b, t_1} \; U^c(X^a, P, t_1) = \\ &\int_0^{t_1} \left(\ln\left(X^a(A_1^a - 1)\right) + \ln\left(X^b(A_1^b - 1)\right) - (\beta^a + \beta^b)P\right) e^{-\rho t} dt \\ &+ \int_{t_1}^{T} \left(\ln\left(X^a(A_2^a - 1)\right) + \ln\left(X^b(A_2^b - 1)\right) - (\beta^a + \beta^b)P\right) e^{-\rho t} dt \end{aligned} \tag{48}$$

subject to :

$$\dot{P} = \alpha_2^a A_2^a X^a(t) + \alpha_2^b A_2^b X^b(t), \quad \text{if} \quad t \geq t_1, \tag{49}$$

$$\dot{P} = \alpha_1^a A_1^a X^a(t) + \alpha_1^b A_1^b X^b(t), \quad \text{if} \quad t < t_1 \tag{50}$$

where P_0 given and $P(T)$ free. We shall solve the above problem starting from $[t_1, T]$ as in the previous cases. Again, we will skip the intermediate easy but cumbersome algebra.

Control on $[t_1, T]$. The Hamiltonian corresponding to the new technology regime is

$$\begin{aligned} H_2 = &-e^{-\rho t}\left(\ln(X^a(A_2^a - 1)) + \ln(X^b(A_2^b - 1)) - (\beta^a + \beta^b)P\right) \\ &+ \lambda_2(\alpha_2^a A_2^a X^a(t) + \alpha_2^b A_2^b X^b(t)) \end{aligned}$$

and the corresponding first-order conditions are:

$$\lambda_2 \alpha_2^a A_2^a = \frac{e^{-\rho t}}{X^a} \tag{51}$$

$$\lambda_2 \alpha_2^b A_2^b = \frac{e^{-\rho t}}{X^b} \tag{52}$$

$$\dot{\lambda}_2 = -e^{-\rho t}(\beta^a + \beta^a) \tag{53}$$

$$\dot{P} = \alpha_2 A_2^a X^a + \alpha_2^b A_2^b X^b\,. \tag{54}$$

This canonical system of equations can be solved using the usual transversality condition and the pollution level at t_1, $P(t_1)$. Consequently, we obtain the following solution path of the co-state variable

$$\lambda_2 = \frac{(\beta^a + \beta^b)}{\rho}(e^{-\rho t} - e^{-\rho T}).$$

The corresponding paths for pollution, production inputs and the optimal value of Hamiltonian at t_1 follow:

$$P = \frac{-2}{(\beta^a + \beta^b)} \ln\left(e^{-\rho t} - e^{-\rho T}\right) + \tilde{p},$$

$$X^a(t) = \frac{1}{\alpha_2^a A_2^a} \cdot \frac{\rho}{\beta^a + \beta^b} \cdot \frac{1}{1 - e^{-\rho(T-t)}}$$

$$X^b(t) = \frac{1}{\alpha_2^b A_2^b} \cdot \frac{\rho}{\beta^a + \beta^b} \cdot \frac{1}{1 - e^{-\rho(T-t)}}\,,$$

$$H_2^*(P_1, t_1) = e^{-\rho t_1}\left(2 - Q_2 + 2\rho t_1 + (\beta^a + \beta^b)\tilde{p}\right)$$

where $Q_2 = \ln\left(\frac{\rho}{\alpha_2^a A_2^a \beta^a}\right) + \ln\left(\frac{\rho}{\alpha_2^b A_2^b \beta^b}\right) + \ln(A_2^a - 1) + \ln(A_2^b - 1)$. A constant given by the initial condition $P(t_1)$ is $\tilde{p}$:

$$\tilde{p} = P_1 + \frac{2}{(\beta^a + \beta^b)} \ln\left(e^{-\rho t_1} - e^{-\rho T}\right).$$

Control on $[0, t_1]$. The Hamiltonian corresponding to the first technology regime is

$$H_1 = -e^{-\rho t}\left(\ln(X^a(A_1^a - 1)) + \ln(X^b(A_1^b - 1)) - (\beta^a + \beta^b)P\right)$$
$$+\lambda_2(\alpha_1^a A_2^a X^a(t) + \alpha_1^b A_1^b X^b(t)),$$

and the resulting optimality conditions are the same as the previous ones, after allowing for the correct technological parameters.

The continuity condition: $\lambda_1(t_1^-) = \lambda_2(t_1^+)$, allows us to calibrate the optimal path of the co-state variable as follows:

$$\lambda_1 = \frac{(\beta^a + \beta^b)}{\rho}(e^{-\rho t} - e^{-\rho T}).$$

The resulting pollution path, production inputs paths and the Hamiltonian value at t_1 are

$$P(t) = \frac{-2}{(\beta^a + \beta^b)} \ln\left(e^{-\rho t} - e^{-\rho T}\right) + \hat{p},$$

$$X^a(t) = \frac{1}{\alpha_2^a A_2^a} \cdot \frac{\rho}{\beta^a + \beta^b} \cdot \frac{1}{1 - e^{-\rho(T-t)}}$$
$$X^b(t) = \frac{1}{\alpha_2^b A_2^b} \cdot \frac{\rho}{\beta^a + \beta^b} \cdot \frac{1}{1 - e^{-\rho(T-t)}},$$

$$H_1^*(P_1, t_1) = e^{-\rho t_1}\left(2 - Q_1 + 2\rho t_1 + (\beta^a + \beta^b)\tilde{p}\right)$$

where $Q_1 = \ln\left(\frac{\rho}{\alpha_1^a A_1^a \beta^a}\right) + \ln\left(\frac{\rho}{\alpha_1^b A_1^b \beta^b}\right) + \ln((A_1^a - 1)) + \ln(\left(A_1^b - 1\right)^b)$; as before, $\hat{p}$ is a constant determined by $P(0)$. After some more algebra, one gets:

$$P_1(t) = P_0 + \frac{2}{\beta^a + \beta^b} \ln\left(\frac{1 - e^{-\rho T}}{e^{-\rho t_1} - e^{-\rho T}}\right).$$

Existence of t_1. An interior optimizer t_1 has to satisfy the following condition:

$$H_2 - H_1 =$$
$$e^{-\rho t_1}\left(2 - Q_2 + 2\rho t_1 + (\beta^a + \beta^b)\tilde{p}\right) - e^{-\rho t_1}\left(2 - Q_1 + 2\rho t_1 + (\beta^a + \beta^b)\tilde{p}\right) = 0;$$

it leads us to the same condition as before

$$e^{-\rho t_1}(Q_1 - Q_2) = 0.$$

Again, there is no rationale to adopt (jointly) the new technology at an interior date. It is optimal to immediately start using it if $Q_1 < Q_2$ or infinitely postpone, if $Q_1 > Q_2$.

This prompts us to expand $Q_1 > Q_2$ and study the inequality

$$\frac{(\alpha_2^a A_2^a)(\alpha_2^b A_2^b)}{(\alpha_1^a A_1^a)(\alpha_1^b A_1^b)} \gtrless \frac{(A_2^a - 1)(A^b - 1)}{(A_1^a - 1)(A_1^b - 1)}. \tag{55}$$

We notice (55) looks more complicated than (NA) and (IA) and promises a meaningful analysis of the technology adoption process in a coalition of two cooperating countries.

4.2. Discussion

We are particularly interested in the identification of cases where cooperation changes the decision of a country regarding the technology adoption.

We known from Corollary 3 that a country alone, say a, will not adopt the new technology if and only if

$$\frac{\alpha_2^a A_2^a}{(A_2^a - 1)} > \frac{\alpha_1^a A_1^a}{(A_1^a - 1)}.$$

However, under cooperation with country b, country a will start using the technology if

$$\frac{\alpha_2^a A_2^a}{(A_2^a - 1)} \frac{\alpha_2^b A_2^b}{(A_2^b - 1)} < \frac{\alpha_1^a A_1^a}{(A_1^a - 1)} \frac{\alpha_1^b A_1^b}{(A_1^b - 1)}. \tag{56}$$

Therefore, country a will adopt the new technology under cooperation if country b's "new" technology compensates the unfavorable trade-off that a faces, if it acts on its own.

We can rewrite (56) to better examine the difference between the cooperative solution and (IA), which we quote from page 11 as $\dfrac{\alpha_2^a A_2^a}{(A_2^a - 1)} < \dfrac{\alpha_1^a A_1^a}{(A_1^a - 1)}$:

$$\frac{\alpha_2^a A_2^a}{(A_2^a - 1)} < \frac{\frac{\alpha_1^a A_1^a}{(A_1^a - 1)} \frac{\alpha_1^b A_1^b}{(A_1^b - 1)}}{\frac{\alpha_2^b A_2^b}{(A_2^b - 1)}} \quad \Rightarrow \quad \frac{\alpha_2^a A_2^a}{(A_2^a - 1)} < \left(\frac{\alpha_1^b A_1^b}{(A_1^b - 1)} \frac{(A_2^b - 1)}{\alpha_2^b A_2^b} \right) \frac{\alpha_1^a A_1^a}{(A_1^a - 1)}.$$

If country b implemented the new technology then

$$\frac{\alpha_1^b A_1^b}{(A_1^b - 1)} \frac{(A_2^b - 1)}{\alpha_2^b A_2^b} \gg 1. \tag{57}$$

Therefore it is easier for country a to satisfy (56) than (IA).

Cooperation schemes may involve some transfers between countries. Our framework allows implicitly for this phenomenon. In considering (56)-(56), we have seen that country a, which is initially reluctant to adopt the new technology, is willing to do so under cooperation, because it will then benefit from the transfers from the partner country.

On the other hand, country b, which alone would have switched to relatively less efficient technologies (with respect to the cooperative solution), would not accept to do so if it had to support the burden of cooperation with a markedly lagging country, and would only switch to the new technology if the menu allowed it to pay for this burden, see (57).

Of course, this reasoning is relevant for cases of *heterogenous* technological achievements, for example when we deal with a developing and a developed country. However, it is also relevant for the North-North relations since the development of clean technologies is on the top of the R&D priorities of *all* Northern countries. But while it is quite natural to call for technological transfers from North to South, it is more difficult to do so with respect to Western democracies. R&D is costly for all countries, and if an advanced country a does not have access to the most efficient clean technologies, it is typically because it did not invest in the related specific R&D or had no incentive to acquire them. In such a case, technology transfers make little sense.

5. Concluding Remarks

In this paper, we model economic performance *vs.* environmental efficiency trade-offs. For that purpose we use a canonical two-stage optimal-control problem embedded in a game-theoretic structure. We focus on a short-time horizon perspective to make the trade-offs sharper.

Our main result is that, unless cooperating, the countries play selfishly. Having a "dirty" or "clean" neighbor does not affect a country's own decision.

The cooperation problem is difficult as it implies that a "very" good technology is available. Such a technology would have to "compensate" the losses the country, which would have adopted the technology on its own, would incur to convince the neighbor reluctant to implement it on its own. This suggests the future of international agreements and protocols, like Kyoto, is certainly not obvious. If "very" good technologies do not exist and hence cooperative solutions are not implementable, then the countries will adopt the “only” good technologies one by one, if this is optimal for them. The recent Canadian decision of withdrawing from the Kyoto protocol is an illustration of our model predictions. We believe that while our set-up is quite canonical, our results display a degree of reality.

Improvements to our model are obviously possible and include more economic controls (like R&D and pollution abatement policies). An introduction of explicit pollution (state) constraints would help generate internal switching times.* Unfortunately, such model extensions damage the analytical solution characterization. Henceforth, our current and further research involves generation of parameter specific outcomes.

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*However, modeling the damage due to pollution as a *more* convex function of P than in (1) is certain to not give more insight into the new technology adoption process, see [15].

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